Bricks as units of volume

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My goal is to suggest a reconstruction of the calculation of the volume in a mathematical context. In this brief presentation, I'll limit myself to the results, without a lot of details on the argumentation and the evidences (that I gave in some published papers in 2007 and 2008). The most important of the documents providing information on calculation practices are the metrological tables, which were used in the OB period for elementary education of the young scribes. In this presentation, I'll also rely on two mathematical texts from unknown provenance: YBC 4607 and YBC 4708.

1. Metrological tables

Here are sections of metrological tables extracted from Nippur sources (these sections are also attested in other cities where remains of scribal schools were found).

1(diš) gin_2 še 1		**$^1/_2$ še ku_3-babbar 10**		**$^1/_3$ sar a-$ša_3$ 20**	
1(diš) $^1/_3$ gin_2	1.20	1(diš) še	20	$^1/_2$ sar	30
1(diš) $^1/_2$ gin_2	1.30	1(diš) $^1/_2$ še	30	$^2/_3$ sar	40
1(diš) $^2/_3$ gin_2	1.40	2(diš) še	40	$^5/_6$ sar	50
1(diš) $^5/_6$ gin_2	1.50	2(diš) $^1/_2$ še	50	1(diš) sar	1
2(diš) gin_2	2	3(diš) še	1	1(diš) $^1/_3$ sar	1.20
2(diš) $^1/_3$ gin_2	2.20	4(diš) še	1.20	1(diš) $^1/_2$ sar	1.30
2(diš) $^1/_2$ gin_2	2.30	5(diš) še	1.40	1(diš) $^2/_3$ sar	1.40
2(diš) $^2/_3$ gin_2	2.40	6(diš) še	2	1(diš) $^5/_6$ sar	1.50
2(diš) $^5/_6$ gin_2	2.50	7(diš) še	2.20	2(diš) sar	2
3(diš) gin_2	3	8(diš) še	2.40	3(diš) sar	3
etc.		etc.		etc.	

1(diš) šu-si	10	1(diš) šu-si	2
2(diš) šu-si	20	2(diš) šu-si	4
3(diš) šu-si	30	3(diš) šu-si	6
4(diš) šu-si	40	4(diš) šu-si	8
5(diš) šu-si	50	5(diš) šu-si	10
6(aš) šu-si	1	6(diš) šu-si	12
7(diš) šu-si	1.10	7(diš) šu-si	14
8(diš) šu-si	1.20	8(diš) šu-si	16
9(diš) šu-si	1.30	9(diš) šu-si	18
1/3 kuš3	1.40	$^1/_3$ $kuš_3$	20
etc.		etc.	
nam-uš-dagal-la-$še_3$		**nam-sukud-bur_3-$še_3$**	
= for lengths and widths		= for heights and depths	

The metrological tables are a correspondence between measures and sexagesimal place value notation (SPVN).

These tables were learnt by heart during the elementary stage of mathematical education.

There are five metrological tables: for capacities, weights, surfaces, lengths (or other horizontal dimensions), and heights (or other vertical dimensions), always in this order.

The incipit indicates the content of the table for capacities (še = grain), weight (ku_3-babbar = silver), surfaces (a-$ša_3$ = field). The tables for linear dimensions do not have heading in Nippur, but have a subscript in Ur:
Table L: nam-uš-dagal-la-$še_3$=for lengths and widths (UET 7 114 et UET 7-115)
Table H: nam-sukud-bur_3-$še_3$ = for heights and depths (UET 7-115)
As we see, **there is no table for volume**.

2. Three systems for volumes

In YBC 4607, the statement of the problem #1 reads as follows

1 1. sig_4 1/2 $kuš_3$ uš-bi
2. 1/3 $kuš_3$ sag-bi 5 šu-si sukud-bi
3. gagar sahar-bi u_3 i_3-$šam_2$ sahar-bi en-nam
4. 12 še šu-ri-a še gagar-bi 2 še u_3 <igi>-12-gal_2 še sahar-bi
5. 3 1/3 $sila_3$ 8 1/3 gin_2 i_3-$šam_2$ sahar-bi

1 1. A brick. 1/2 $kuš_3$ its length,
2. 1/3 $kuš_3$ its width, 5 šu-si its height.
3. Its base, its volume and its capacity is what?
4. 12 1/2 še its base, 2 1/12 še its volume,
5. 3 1/3 $sila_3$ 8 1/3 gin_2 its capacity.

Data are the length, the width and the height of a brick. It is asked to calculate its volume and its capacity.

In YBC 4708, the statement of the problem #1 reads as follows

1. [sig_4]-⌈anše⌉* 5 ninda uš-bi
2. [1 1/2] ninda sag 1/2 ninda sukud-bi
3. sig_4-bi en-nam
4. sig_4-bi 3(iku) GAN_2 24 sar

1. A pile of bricks. 5 ninda its length,
2. [1 1/2] ninda its width, 1/2 ninda its height.
3. Its (volume in) bricks is what ?
4. Its (volume in) bricks is 3(iku) GAN_2 24 sar.

Data are the length, the width and the height of a pile of bricks (rectangular prism). It is asked to calculate its volume expressed in numbers of bricks.

Conclusion:
There are three manners to calculate a volume:

- Standard volume (sahar)
- Capacity (i_3-$šam_2$ sahar)
- Brick volume (sig_4)

3. Units of volume

The standard units of volume are units of surfaces with a constant height of 1 $kuš_3$ (50 cm). In the table of vertical dimensions, 1 $kuš_3$ corresponds to 1. As a consequence, the table of surface can be used as a table of volume.

Surface units (1 sar ≈ 36 m²) and **volume units** (1 sar ≈ 18 m^3)

GAN_2	←100—	sar	←60—	gin_2	←180—	še
						
1.40		1		1		20

4. Calculating the volume

Let us come back to YBC 4607 #1.

1 1. sig_4 1/2 $kuš_3$ uš-bi
2. 1/3 $kuš_3$ sag-bi 5 šu-si sukud-bi
3. gagar sahar-bi u_3 i_3-$šam_2$ sahar-bi en-nam
4. 12 še šu-ri-a še gagar-bi 2 še u_3 <igi>-12-gal_2 še sahar-bi
5. 3 1/3 $sila_3$ 8 1/3 gin_2 i_3-$šam_2$ sahar-bi

1 1. A brick. 1/2 $kuš_3$ its length,
2. 1/3 $kuš_3$ its width, 5 šu-si its height.
3. Its base, its volume and its capacity is what?
4. 12 1/2 še its base, 2 1/12 še its volume,
5. 3 1/3 $sila_3$ 8 1/3 gin_2 its capacity.

The measures of bricks are converted into SPVN using metrological tables

Table L

9 šu-si	1.30
$^1/_3$ $kuš_3$	1.40
$^1/_2$ $kuš_3$	2.30
$^2/_3$ $kuš_3$	3.20

Table H

4 šu-si	8
5 šu-si	10
6 šu-si	12

Table S

1/12 še	1.40
[…]	
¼ še	5
1/3 še	6.40
½ še	10
2/3 še	13.20
5/6 še	16.40
1 še	20
2 še	40

Length: 1/2 $kuš_3$ → 2.30 (table L)
Width: 1/3 $kuš_3$ → 1.40 (table L)
Height: 5 šu-si → 10 (table H)

The base is: 2.30 × 1.40 = 4.10
The volume is: 74.10 × 10 = 41.40

The corresponding measure of volume is obtained by a reverse reading of table S (completed with the section for small dimensions):

41.40 → 2 1/12 še

5. Brick-volume

The use of bricks as unit of volume is attested as soon as the end of the third millennium. Vincent Scheil (1915, Le calcul des volumes dans un cas particulier à l'époque d'Ur, RA 12 : 161-72) shown that the units of the surface system were used four counting bricks.

In the brick system: 1 sar = 720 bricks whatever the size of the bricks.

Surface units (1 sar ≈ 36 m²), **volume units** (1 sar ≈ 18 m^3) and **brick numbers** (1 sar ≈ 720 bricks)

GAN_2	←100—	sar	←60—	gin_2	←180—	še
						
1.40		1		1		20

The volume in the brick system is the number of bricks expressed in sar (or multiple or sub multiple of the sar).
As 1 sar of bricks is always 720 bricks whatever the size of the bricks, the volume in the brick system depends on the size of the bricks.

The bricks used as units of volume in mathematical texts form a small set of five types. The dimensions of these standardized bricks are given in tablet YBC 4608:

#	**Name of the brick**	**Dimensions of the brick**
1	sig_4	1/2 $kuš_3$; 1/3 $kuš_3$; 5 šu-si
2	sig_4	18 šu-si ; 12 šu-si ; 5 šu-si
3	sig_4-ab_2	2/3 $kuš_3$; 1/3 $kuš_3$; 5 šu-si
4	sig_4-al-ur_3-ra	2/3 $kuš_3$; 2/3 $kuš_3$; 5 šu-si
5	sig_4-al-ur_3-ra	1 $kuš_3$; 1 $kuš_3$; 5 šu-si

6. The nalbanum

The *nalbanum* (or "brickage" in MCT: 138) is a coefficient expressing the relationship between the volume in bricks (sar-brick) and the standard volume (sar-volume). Each type of brick has its *nalbanum.* This coefficient can easily be calculated knowing the dimensions of the bricks. The *nalbanum* is also given in tables of coefficients, for example in BM 36776 (see Robson 1999: 59, 206, 207). The *nalbanum* of the 5 types of bricks of YBC 4607 are the following:

	Name of the brick	Dimensions of the brick	Standard olume V	*nalbanum* = 5/V
1	sig_4	1/2 $kuš_3$; 1/3 $kuš_3$; 5 šu-si	2.30×1.40×10 = 41.40	7.12
2	sig_4	18 šu-si ; 12 šu-si ; 5 šu-si	3×2×10 = 1	5
3	sig_4-ab_2	2/3 $kuš_3$; 1/3 $kuš_3$; 5 šu-si	3.20×1.40×10 = 55.33.20	5.24
4	sig_4-al-ur_3-ra	2/3 $kuš_3$; 2/3 $kuš_3$; 5 šu-si	5.20×3.20×10 = 2.57.46.40	2.42
5	sig_4-al-ur_3-ra	1 $kuš_3$; 1 $kuš_3$; 5 šu-si	5×5×10 = 4.10	1.12

By multiplying the standard volume by the *nalbanum*, one can read the number in of bricks (expressed in measure of brick-volume) directly in the table S.

7. Calculating the bricks-volume

Let us come back to YBC 4708 #1. The dimensions of a pile of bricks are given, and the brick-volume of the pile is requested.

1 A pile of bricks. 5 ninda its length,
[1 1/2] ninda its width, 1/2 ninda its height.
Its (volume in) bricks is what ?
Its (volume in) bricks is 3(iku) GAN_2 24 sar.

The type of the bricks is not précised, so one guesses that it is the most common brick, that is the first type (*nalbanum* 7.12).

Table L

$^1/_2$ ninda	30
1 ninda	1
1 ½ ninda	1.30
2 ninda	2
3 ninda	3
4 ninda	4
5 ninda	5

Length: 5 ninda → 5
Width: 1 1/2 ninda → 1.30
Height: 1/2 ninda → 6

Base: 5 × 1.30 = 7.30
Standard volume: 7.30 × 6 = 45
Brick volume: 45 × 7.12 = 5.24

Table H

4 $kuš_3$	4
5 $kuš_3$	5
$^1/_2$ ninda	6
1 ninda	12

Table S

4 sar	4
5 sar	5
[…]	
10 sar	10
20 sar	20
30 sar	30
40 sar	40
1(ubu) GAN_2	50
1(ubu) GAN_2 1sar	1
3(iku) GAN_2	5

Measure of brick-volume:

5.24 → 3(iku) GAN_2 24 sar