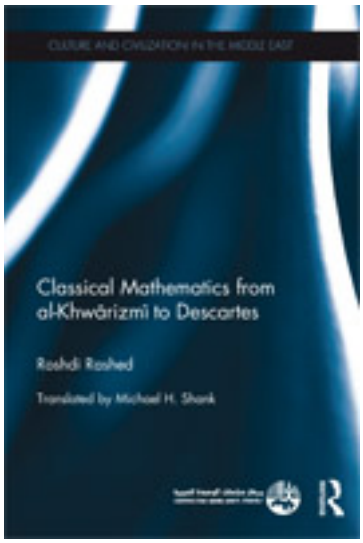
Classical Mathematics from al-Khwārizmī to Descartes



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Translated by Michael Shank

This book follows the development of classical mathematics and the relation between work done by the Arab and Islamic worlds and undertaken by the likes of Descartes and Fermat.
'Early modern' mathematics is a term widely used to refer to the mathematics in the West during the sixteenth and seventeenth century. For many historians and philosophers this the watershed which marks a radical departure from 'classical mathematics', to more modern mathematics ; heralding the arrival of algebra, geometrical algebra, and the mathematics of the continuous. In this book, Roshdi Rashed demonstrates that 'early modern' mathematics is actually far more composite than previously assumed, with each branch having different traceable origins which span the millenium. Going back to the beginning of these parts, the aim of this book is to identify the concept and practices of key figures in their development, thereby presenting a fuller reality of these mathematics.
This book will be of interest to students and scholars specialising in Islamic science and mathematics, as well as to those with an interest in the more general history of science and mathematics and the transmission of ideas and culture.

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