

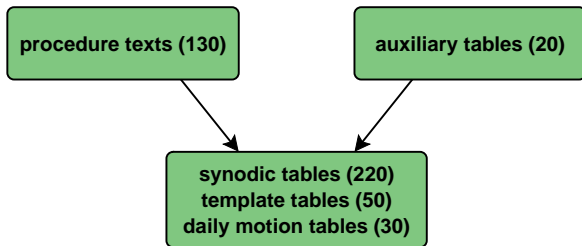
mathematical terminology in Babylonian astronomical texts

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Babylonian mathematical astronomy: text groups

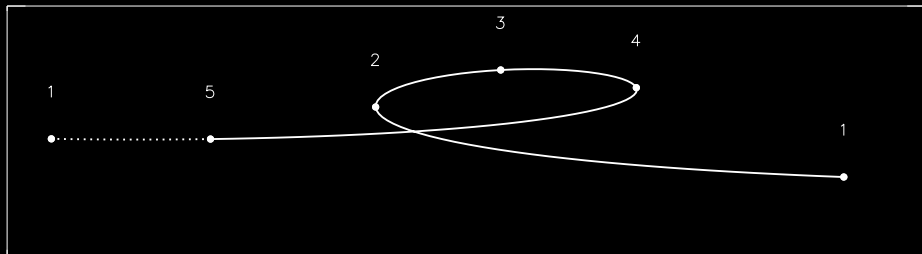


synodic phenomena

Moon: lunations \Rightarrow Lunar Six intervals, eclipses



planets: first and last appearances; stations



purpose of the astronomical algorithms

- updating or computing a function
 - (1) updating a function by applying a difference to the previous value, $f_i = f_{i-1} \pm d$.
 - (2) computing one function from another function, $g(f)$ [$\Rightarrow g$ updated indirectly]
- verification, i.e. by means of *net differences* $df(s)$, where $f_{i+s} = f_i + df(s)$
- other purposes

Babylonian astronomical procedures: previous translations

DIE

BABYLONISCHE MONDRECHNUNG.

ZWEI SYSTEME DER CHALDÄER
ÜBER DEN LAUF DES MONDES UND DER SONNE.

AUF GRUND MEHRERER VON J. N. STRASSMAIER S. J. COPIRTEN KEILINSCHRIFTEN
DES BRITISCHEN MUSEUMS

VON

FRANZ XAVER KUGLER S. J.

MIT EINEM ANHANG: ÜBER CHALDÄISCHE PLANETENTAFELN.

FREIBURG IM BREISGAU.
HERDER'SCHE VERLAGSHANDLUNG.

1900.

ZWEIGNIEDERLASSUNGEN IN WIEN, STRASSBURG, MÜNCHEN UND ST. LOUIS, MO.

Sources
in the History of Mathematics and
Physical Sciences 5

O. NEUGEBAUER

ASTRONOMICAL CUNEIFORM TEXTS I

Springer-Verlag New York Heidelberg Berlin

Babylonian astronomical procedures: previous translations

Kugler (1900), 'Die Babylonische Mondrechnung':

Neugebauer (1938), 'Untersuchungen zur antiken Astronomie II', *Quellen und Studien B* 4, 34–91:

'Here [in Babylonia], for the first time in the history of mankind, one has succeeded in controlling (beherrschen) the laws of a very complicated natural phenomenon through purely mathematical methods.'

Neugebauer (1946), 'History of Ancient Astronomy: Problems and Methods', *PASP* 58, 17–43, 104–142:

'For methodological reasons it is obvious that a drastic restriction in terminology must be made. We shall here call 'astronomy' only those parts of human interest in celestial phenomena which are amenable to mathematical treatment.'

Neugebauer (1955), Glossary to *ACT* (p. 467):

'The translations given are not intended to be strictly literal but rather try to convey the general meaning, especially for technical terminology.'

example: function F (Moon system A)

F = Moon's daily displacement along the zodiac

column F in synodic table ACT 18 (Babylon 1st c. BCE):



14;13,35,37,30

14;55,35,37,30

15;37,35,37,30

15;34,13,7,30

14;52,13,7,30

14;10,13,7,30

13;28,13,7,30

12;46,13,7,30

12;04,13,7,30

11;22,13,7,30

11;27,56,15

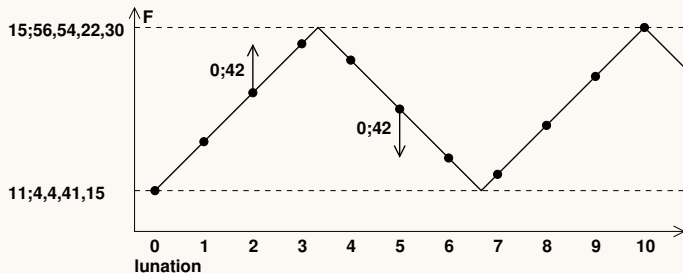
12;09,56,15

12;51,56,15

BMAPT No. 53 P7'.a: updating F as a zigzag function

^{14'} *epūš(du₃-uš) ša₂ nisiḥ(zi) Sin arḥa(ab₂) ana arḥi(ab₂) 42 teṭepi(tab) u tumaṭṭa(la₂) lib₃-bu-u₂ ša₂ 15.56.54.22.30 takaššad(kur-ad₂)* ^{15'} *ša₂ al 15.56.54.22.30 atru(diri) ina(ta) 15.56.54.22.30 tanaḥḥas(la₂) lib₃-bu-u₂ ša₂ 11.4.4.41.15* ^{16'} *takaššad(kur-^rad₂) ša₂ al 11.4.4.41.15 maṭû(la₂-u₂) itti(ki) 11.4.4.41.15 teṭepi(tab)*

^{14'} Procedure for the displacement of the Moon. Month by month you add and subtract 0;42, whereby you reach 15;56,54,22,30. ^{15'} (The amount) by which it exceeds 15;56,54,22,30 you subtract from 15;56,54,22,30, whereby you ^{16'} reach 11;4,4,41,15. (The amount) by which it is less than 11;4,4,41,15 you add with 11;4,4,41,15.



mathematical terminology: preliminary remarks

procedures and algorithms

- procedure = verbal representation of an algorithm
- algorithm = *complete* sequence of mathematical operations for computing a quantity, reconstructed from procedure texts and/or tabular texts

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elements of a procedure

- introduction (statement of purpose)
- arithmetical operations
- conditions: involving thresholds, direction of change, or orientation ('if it is larger/smaller'; 'increasing/decreasing'; 'above/below')
- storage of data ('you put it down ...')
- extraction of data ('you hold in your hands ...')
- coordination ('and then...'; '... which you had put down')
- naming ('you call it ...'; 'you put it down as ...')

arithmetical operations: addition

to add ('append') x to|with y

x itti(**ki**)|ana y ʔepû(**tab**)

LB

BMAPT No. 61.A Obv. 19:

Moon system A

nishā(zī) ša₂ sin issuhū(zī) itti(ki)
qaqqari(ki) sin teʔeppi(tab)

The distance by which the Moon moved you
add ('append') with the position of the Moon.

⇒ asymmetric, identity-conserving addition

arithmetical operations: addition

to add ('append') x **to|with** y x *itti(ki)|ana y tepû(tab)* LB

BMAPT No. 61.A Obv. 19: Moon system A

nishā(zi) ša₂ sin issuhū(zi) itti(ki) The distance by which the Moon moved you
qaqqari(ki) sin teṭeppi(tab) add ('append') with the position of the Moon.

⇒ asymmetric, identity-conserving addition

to append x **and** y **together** x u y {*itti(ki)|ana muḥḥi*} *aḥāmiš tepû(tab)* LB

BMAPT No. 61.D Obv. 3: Moon system A

gi₆ du u LA₂ ša₂ 'me' itti(ki) a-ḥa-miš₂ (The time by which) the night has progressed
teṭeppi(tab) (M) and the length of 'daylight' (C) you add
together.

⇒ asymmetric addition, loss of identity

arithmetical operations: addition

to accumulate x and y together

x u y {*itti*(**ki**)|*ana muḥḥi*} *aḥāmiš kamāru*(GAR.GAR)

OB-LB

BMAPT No. 53 Rev. ii8':

ana muḥḥi(**ugu**) *a-ḥa-miš*₂ *ša*₂ *sin* u
šam[*aš*₂ *takammar*(GAR).GAR]-*ma* $\frac{1}{2}$ -
*šu*₂ *tanašši*(GIŠ)

You accumulate (the coefficients) for the Moon and the Sun together and you compute half of it.

⇒ symmetric addition, loss of identity

arithmetical operations: subtraction and multiplication

subtractive operations

to 'tear out' x from y

x *ina*(**ta**) y *nasāḥu*(**zi**)

OB-LB

to deduct ('raise') x from y

x *ina*(**ta**) y *šūlû*(**nim**, **e**₁₁)

LB

to subtract x from y

x *ina*(**ta**) y *naḥāsu*(**la**₂)

LB

to reduce y by x

x *ana* y *muṭṭû*(**la**₂)

LB

arithmetical operations: subtraction and multiplication

subtractive operations

to 'tear out' x from y	x <i>ina</i> (ta) y <i>nasāḥu</i> (zi)	OB-LB
to deduct ('raise') x from y	x <i>ina</i> (ta) y <i>šūlû</i> (nim , e ₁₁)	LB
to subtract x from y	x <i>ina</i> (ta) y <i>naḥāsu</i> (la ₂)	LB
to reduce y by x	x <i>ana</i> y <i>muṭṭû</i> (la ₂)	LB

multiplication

to multiply ('go') x times y	x a.ra ₂ GAM GAM ₀ y <i>alāku</i> (du)	OB-LB
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BMAPT No. 61.A Obv. 11:

Moon system A

birīta(**bi**₂) GAM₀ *šilīpti*(**bar.nun**) *tal-*
lak(**du**)

You multiply the elongation by the *šilīptu*-
coefficient.

arithmetical operations: subtraction and multiplication

subtractive operations

to 'tear out' x from y	x <i>ina</i> (ta) y <i>nasāḥu</i> (zi)	OB-LB
to deduct ('raise') x from y	x <i>ina</i> (ta) y <i>šūlû</i> (nim , e ₁₁)	LB
to subtract x from y	x <i>ina</i> (ta) y <i>naḥāsu</i> (la ₂)	LB
to reduce y by x	x <i>ana</i> y <i>muṭṭû</i> (la ₂)	LB

multiplication

to multiply ('go') x times y	x a.ra ₂ GAM GAM ₀ y <i>alāku</i> (du)	OB-LB
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BMAPT No. 61.A Obv. 11: Moon system A

birīta(**bi**₂) **GAM**₀ *šilipti*(**bar.nun**) *tal-lak*(**du**) You multiply the elongation by the *šiliptu*-coefficient.

very rare:

x a.ra ₂ <i>ana</i> y <i>našû</i> (il ₂)	to 'raise' x times to y	OB-LB
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arithmetical operators: division and reciprocals

new terms for division

x ana y aḥḥê(šēš.meš) zâzu(bar , SE ₃)	To divide x into y parts.	LB
n -š _u ₂ našû(GIŠ)	To compute ('raise') $1/n$ of it.	LB

reciprocal (very rare in the procedure texts):

igi x gal ₂ .bi y	The reciprocal of x is y .	OB-LB
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additive and subtractive numbers: innovations

1. additive/subtractive numbers isolated from their application

$\times \text{tab}/\text{la}_2 = 'x \text{ add/subtr.}'$

example: column J (Moon system A)



[57.3.4]5 la_2

[57.3.4]5 la_2

57.3.45 la_2

57.3.45 la_2

57.3.45 la_2

27.53.50

[57;3,4]5 subtr.

[57;3,4]5 subtr.

57;3,45 subtr.

57;3,45 subtr.

57;3,45 subtr.

27;53,50

2.32.10 la_2

57.3.45 la_2

2;32,10 subtr.

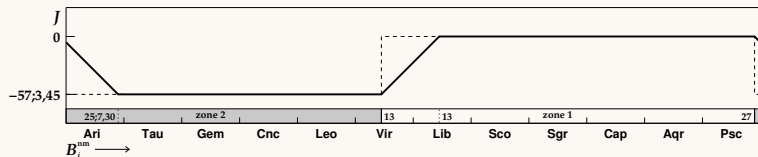
57;3,45 subtr.

additive and subtractive numbers: innovations

2. arithmetical operations on additive/subtractive numbers

example 1: J = zodiacal correction to synodic month modeled as step function of Moon's zodiacal position (B)

Moon system A



$$J_i = \begin{cases} -57;3,45^\circ + 1;54,7,30 \cdot (B_i - 13^\circ \text{ Vir}) & (13^\circ \text{ Vir} \leq B_i \leq 13^\circ \text{ Lib}) & (1a) \\ 0^\circ & (13^\circ \text{ Lib} \leq B_i \leq 27^\circ \text{ Psc}) & (1b) \\ -2;1,44 \cdot (B_i - 27^\circ \text{ Psc}) & (27^\circ \text{ Psc} \leq B_i \leq 25;7,30^\circ \text{ Ari}) & (2a) \\ -57;3,45^\circ & (25;7,30^\circ \text{ Ari} \leq B_i \leq 13^\circ \text{ Vir}). & (2b) \end{cases}$$

BMAPT No. 53 P14', interval 2a:

From 27 Psc until 25;7,30 Ari [you subtract] 2;1,44 for 1° [from the duration.] [...]

You multiply 2;1,44, subtractive (2.1.44 $\mathbf{1a}_2$), by 28;7,30, the position of the Sun, (it is 57;3,45, subtractive (57.3.45 $\mathbf{1a}_2$)).

2. arithmetical operations on additive/subtractive numbers

example 2: correcting additive/subtractive corrections (lunar system K)

BMAPT No. 52 Oi35 ('atypical Text K') Obverse i35, Babylon 4rd c. BCE

You 'tear out' (**zi** = *tanassah*) 0;32 from 22, the subtraction (**zi** = *nishu*) for Ari, it is 21;28.

- 'subtractions' are not negative quantities but positive ones to be subtracted
- no evidence of sign rules (e.g. subtractive number times subtractive number)

formulation of conditions

1. involving a threshold

ki-i Q al-la Q₀ atru(diri) ...

If *Q* exceeds *Q₀* ...

ki-i Q al-la Q₀ maṭû(la₂)|i-ṣi ...

If *Q* is less|smaller than *Q₀* ...

2. involving change or relative position

ki-i ṭepû(tab) ... ki-i maṭû(la₂) ...

If it is increasing ... if it is decreasing ...

ki-i šaqû(nim, la₂) ... ki-i šaplu(sig, bur₃) ...

If it is 'high' ... if it is 'low' ...

ki-i išaqqû(nim, la₂) ... ki-i išappilu(sig, bur₃) ...

If it is ascending ... if it is descending ...

terminological innovations: 'functional' concepts

differences

difference = *tašpiltu(taš)* [SN *ana* SN], 'difference [(from) SN to SN]';

or: *birīt* SN *ana* SN, 'distance [(from) SN to SN]'

of incr/decr. function: *tīpu(tab) u mītu(la₂)*, 'addition and subtraction'

extrema and mean values

minimum = *šeḫertu(tur)*, 'the smallest one'; *šuplu(sig)*, 'depth'

maximum = *rabītu(gal)*, 'the largest one'; *šūqu(nim, la₂)*, 'height'

mean = *qablû(murub₄-u₂)*, 'middle'; *kajjamānu(sag.uš)*, 'steady'

template procedures

BMAPT No. 53 P7'.a:

d ab₂ ana ab₂ tab u la₂

ša₂ al-la M diri ta M la₂

ša₂ al-la m la_{2-u₂} ki m tab-ma gar

Month by month you add and subtract *d*.

(The amount) by which it exceeds *M* you subtract from *M*.

(The amount) by which is less than *m* you add with *m* and put down.

another template:

d tab u la₂

[en *M gal*] *ša₂ al-la M diri|gal-u₂ ta 2M la₂|e₁₁|nim[-ma gar]*

[en *m tur*] *ša₂ al-la m tur-er ta 2m la₂|e₁₁|nim[-ma gar]*

You add and subtract *d*.

[Until *M*, the largest one;] that which exceeds|is larger than *M* you subtract|deduct from *2M* [and put down].

[Until *m*, the smallest one;] that which is smaller than *m* you subtract|deduct from *2m* [and put down].

BMAPT No. 13 P11'.a (Mars System A)

mi-nu-u₂ a.ra₂ 2.13 tāmarāti(igi.meš) lullik(lu-du)-ma lu-^ru₂ 15.6 6.48.43.18.30 GAM
2.13 igi.meš tallak(du)-ma

What should I multiply by 2,13 appearances so that it is 15,6,0? You multiply 6,48;43,18,30 by 2,13 appearances, it is 15,6,0.

aššu(mu) lā(nu) tīdû(zu-u₂): ...

Since you do not know it: you ...

reconstructed rhetorical model of an astronomical procedure:

- 1 What should I? / How do I ...?
- 2 Since you do not know it:
- 3 Procedure for You put down ..., you, you call it ...

features shared with OB mathematical problem texts

- sexagesimal number system
- part of the arithmetical terminology
- semantic differentiation (mainly for addition) regarding
 - loss/conservation of identity of involved quantities
 - symmetry/asymmetry of the operation

number notation: for vanishing digits (0) and vanishing number ('nothing'; 'it does not exist'; empty space)

arithmetical terminology

- most terms are replaced (in particular most of the OB 'geometrical algebra')
- 'general' terms replacing (partly) object-specific terms of OB mathematics
- division virtually absent except trivial divisions ($1/n$)
- use of u , 'and', as place-holder for arithmetical operations in references to a known result

invention of terminology to cover new mathematical concepts:

- additive/subtractive numbers:
 - 1 \times **tab**/**la**₂: additive/subtractive numbers isolated from application $y \pm x$
 - 2 subjected to *all* 3 basic arithmetical operations with bare numbers x
 - 3 of undetermined magnitude: *tīpu*(**tab**) = 'addition' ('appendum'); *mītu*(**la**₂) = 'subtraction' ('lack'); *nishu*(**zi**), 'that which is torn out'
- for addressing change of quantities (increasing/decreasing)
- for addressing 'functional concepts':
 - minimum, maximum, mean value
 - difference, net difference for s cycles
 - period relations, number period

Moon system A

