Mathematical Computations in the Management of Public Work in Mesopotamia (End of the 3rd and Beginning of the 2nd Millennium BCE).

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ABSTRACT

1. INTRODUCTION

Architecture is one of the very few fields of research within Mesopotamian studies where it is possible to link together archaeological and ethnographic data (and, to a smaller extent, iconography) with the huge information provided by numerous cuneiform tablets unearthed till more than one century and a half. Ancient Mesopotamian architecture is an earthen architecture using mainly sundried and baked bricks with earthen mortar and plasters and almost no stones. The making process of these bricks and of this architecture is rather well known; it has not changed at all until very recently (i.e. the end of the 20th century with the arrival of concrete materials) and thus ethnographical data are very valuable beside the archaeological ones. The numerous Mesopotamian texts indicate that theses making processes were the same than now.
The aims of my present communication is to see what sort of calculations the Mesopotamian scribes were trained at in the domain of building activities and to try to check how far these calculation methods were really used in practise both with the administrative texts dealing with construction activities and with the archaeological and ethnographical data.

I'll restrain my present study to the period of the end of the third millennium and the beginning of the second millennium BC, which is a period where we can find both numerous mathematical and administrative texts together with important archaeological excavations of public buildings. The Third Dynasty of Ur lasted a century between 2100 and 2000. The five kings of the dynasty conducted a very ambitious construction policy, especially in the religious domain. During this period, the first Mesopotamian ziggurats have been built, surrounded with very important religious complexes. These buildings required the making and the laying of millions of bricks. Only a highly developed bureaucracy could do this. During this period the classical metrological Mesopotamian system has been completed. The first steps seem to have began in the middle of the third millennium and to have been improved by the Akkadian rulers. These standards were also in use during the OB period at the beginning of the second millennium BC and most of them remained in use until the end of the first millennium BC, i.e. the late Babylonian period.

Two sorts of texts provide our information about mathematical calculations in the building process. First, mathematical texts, which are both references tables as metrological tables, and scribes exercises.
Second, administrative or practical texts, which can be provisional estimations of the material and the workers needed or a construction project or statements of account of the raw material used or of the total wages paid to the workers, with list of workers and their task.

2. MATHEMATIC TEXTS DATA

2.1. Constants

The first item is constants, in this category we can put the daily tasks asked to the unskilled workers, these daily tasks were named *iškarum.*
We have here only one variable: the total volume of building material to be processed is divided by the daily task in order to find the total number of days necessary.

$$\frac{total\, volume\ or\ surface}{daily\ task} \rightarrow numbers\ of\ days\, of\ work$$

These constants are very important for archaeologists and historians because they give us an idea of the work charge asked to unskilled workers in ancient Mesopotamia. We can compare them with ethnographic information and note then that they were realistic values. Furthermore, these *iškarum* constants may be found in both mathematical and administrative texts, which means that they were actually used.

2.2. Coefficients
The second step is the use of coefficients to translate one value to another for the calculation (volume to weight for instance).

2.2.1. Transport
The main example for construction activities is the case of material carrying. In this case two variables are used: the volume of the material to be carried and the distance. The load was named after the basket *tupsikkum*. It is difficult to determine its precise value, I am not sure if it was a constants or it may be different in regard of the material (for instance, it may be easier to transport reeds than water for the same weight and the load could thus have been different). It seems that the load was varying between 25 and 50 kg. These are realistic values; a modern cement sack weights 50 kg.
Calculations were in fact not done with the weight of the material to be carried but with its volume. The scribes had then to apply a coefficient to the volume so as to find the weight and to calculate the number of loads.

$$\frac{earth\,carrying\,coefficient}{distance\,of\ each\,trip} \to volume\,of\ earth\,carried\,in\,a\,day$$

The calculation of the days of work depends then on the distance, it was made wth another coefficient named *mutalikum* « the going ». The daily rate of carrying was calculate as follows:

$$\frac{going(ninda\,/\,day) \times basket(sar_v)}{distance(ninda)} \to daily\,rate\,of\ carrying(sar_v\,/\,day)$$

2.2.2. Bricks
Calculation of days of work is a little bit complicated when carrying bricks because the load is calculated in this case as a number of bricks and not a volume, this number depending on the weight of one bricks and so on its dimensions. There were of course several types of bricks of various dimensions.
Mathematical texts give us evidence for these different types of bricks. Yale Tablet YBC 4708, for instance, gives 5 different types, Powell and then Robson and Friberg have added some other up to a total of 14 types. The Yale type one or Powell *et alli* type 2 is a rectangular bricks measuring half a cubit of length (i.e. 25 cm) for 2/3 cubit width (about 17 cm) and 5 fingers thickness (8 cm) that is to say proportions of 1, 1/3 and 2/3. It is the most common unbaked (that is to say sundried) brick used during Ur III and OB period, very well attested in the archaeological data. The Yale type 4 is a square brick measuring ½ a cubit on 5 fingers thickness. It is a baked brick (*agurrum*) as the type 5 measuring 1 cubit. This last type is more probably a theoretical type than an actual brick type because we found very rare evidence of these dimensions on the archaeological material. On the archaeological material, we can see that the main bricks types correspond more or less to the most common bricks, that is to say type 2 (Yale type 1) and type 8 (Yale type 3). The dispersion around the theoretical values of the archaeological material is due to the shrink-wrapping of the brick during the drying process or the baking.

There were thus specific coefficients for bricks:
The *nalbanum* or « brickage » is a coefficient used to calculate the number of bricks of a specific type contained in a volume unit:

Volume (in sar_v) x brickage = number of bricks (in sar_b)

This coefficient was used for instance to calculate the number of bricks contained in a brick pile. This was probably a very common task devoted to the scribes.

The *nazbalum* or « carriage » was used to calculate the number of bricks contained in a carrying load. It is proportional to the *nalbanum*.
The average load seems to have been of 6 type 2 bricks: a weight of 50 pounds “mana” (or 25 kg).

Thus, to calculate the number of days necessary to carry a volume of bricks from one point to the other, the scribe had first to convert the total volume in a brick number (with the *nalbanum*), then to calculate the number of loads with the *nazbalum*, and the number of trips with the *muttalikum.*

There was another method for brick counting involving the *taddîtum* or bricklayer: that is to say the number of one type of bricks in a sar area (i.e. 36 m^2). This is a metrological term different from the architectural one *tibkum*, which means a layer of bricks in a wall.
Mathematical exercises give us examples of calculation of the volume of walls according to the surface of a building, a ratio of 1/3 was applied: one third of the complete surface of the house was considered to be occupied by the walls. This ratio is coherent with the archaeological data: in the case of the OB houses of Ur for instance the ratio was varying from 21,4 to 50 % (if there was a storey the walls of the first floor was thicker).

Wall coefficient = 5/6 x brickage
Wall floor surface = 1/3 total surface of the house
1/3 x surface of the house x height of the walls = volume of the walls (ninda^3)
Length of the wall x width x height x brick coefficient [x 5/6] = total brick number

When calculating the number of bricks in a brick pile the *nalbanum* was directly used that is to say that the scribe considered that there were no space between the bricks, but when calculating the numbers of bricks contained in a wall, 1/6 of the total volume was subtracted because of the mortar (the total volume was multiplied by 5/6). The volume of mortar necessary for a construction was calculated probably the same way.

3. Combinating problems
The last sort of problem are the exercises dealing with several tasks. For instance a worker have to carry earth on a distance during a part of the day and to mould bricks with this earth during the rest of the day. In practice workers had probably to assume different tasks in the same day.

CONCLUSIONS

To conclude, we have seen that the OB mathematical texts were using constants that we can find also in administrative texts at last as earlier as the Ur III period and probably till the Akkadian period. That means that mathematical exercises were made with realistic values in order to provide a practical entertainment to the scribes who will work later on building operations. Their work will be both to prepare provisional accounts of materials and wages for a construction project and to calculate the actual wages in regard of the work really done.
On the other hand it is clear that part of theses mathematical exercises, if they use the same realistic data, were not devoted to a practical entertainment but to a theoretical one because the operation or the calculation won't have any sense practically.
If we try to imagine the use of these calculations on a public building operation we encounter several problems. One of them is the fact that we can't find in the mathematical problems any mention of the daily tasks for women, and we know (for instance with the worker lists) that they were also employed for unskilled task like brick carrying. Did they have to carry the same loads and the same wages (they were usually different from those of the men in the administrative wages accounts)? Brick carrying accounts like those from Garshana were more probably brick delivering lists, the wages were in this case calculated not according to the time of work but to a unit value of the material delivered. This could also have been the case for instance for reeds or mats delivering.
One last problem is the one of the definition of each daily task: what was the possibility to change them according to specific situation (for instance carrying bricks on the summit of a ziggurat was calculate at the same rate than on a flat ground?). We know, thanks to a Late Babylonian text, that workers could protest against a daily brick making rate too important.
Finally we can say that the mathematical calculations were really used but that, practically, there was probably an important part of variation around the theoretical values and calculations.