Elevated silo, Senegal, protecting against moisture, insects and animals

Seminar SAW: History of Mathematics, History of Economical and Financial Practices

March 9, 2012

"Grain and Time Calculations in 4th Millennium BC Mesopotamia"

Robert K. Englund

Libyan village Qasr al-Haj with Berber granary from the 2nd half of the 12th century. In the middle is the winch used to haul baskets of grain to the 2nd story.

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Luhya grain transportation baskets on sale at Kakamega market in Kenya; if used for flour, their inside surfaces are smeared with cow dung to close the gaps.

Seminar SAW: History of Mathematics, History of Economical and Financial Practices

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"Grain and Time Calculations in 4th Millennium BC Mesopotamia"

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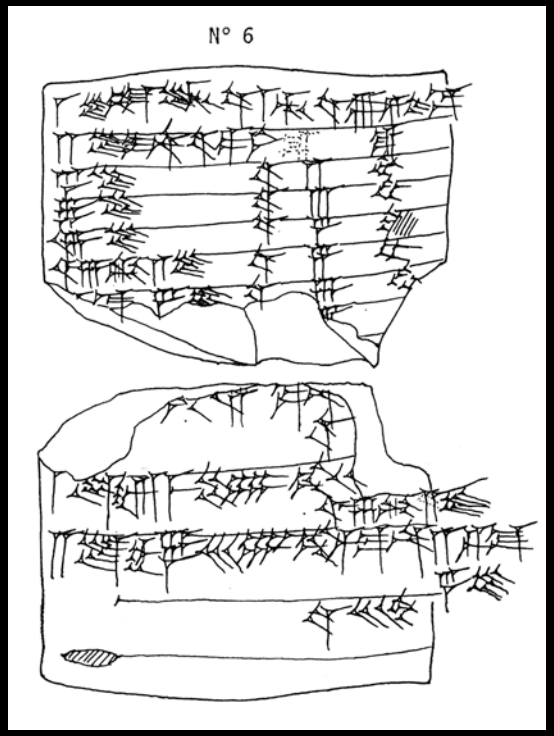
Simple grain measuring and transportation devices included silos at the larger-scale, bags, baskets and pots at the middle, and smaller baskets, boxes and ceramic containers at the smaller scale

Even in the Ozarks of Missouri and Arkansas, the traditional grain scoop is nowadays reduced to candle decoration



P390441 = Denis Soubeyran, *RA* 78, 30-35 6

1 barley-corn: by a single barley-corn I increased,
= approximately 0.05g
2 barley-corns in the 1st day;
4 barley-corns in the 2nd day;
8 barley-corns in the 3rd day;
...
2 'thousand' 7 'hundred' 37 talents 1/2 mina 2 1/3 shekels 4 barley-corns in the 30th day.
= approximately 49,710kg

('lim' used as sexagesimal 600, not decimal 1000;
'me' used as sexagesimal 60, not decimal 100)

Bottomless
wonders spring
from simple rules
… repeated
without end

Benoit Mandelbrot, 1985

Uruk Chronology

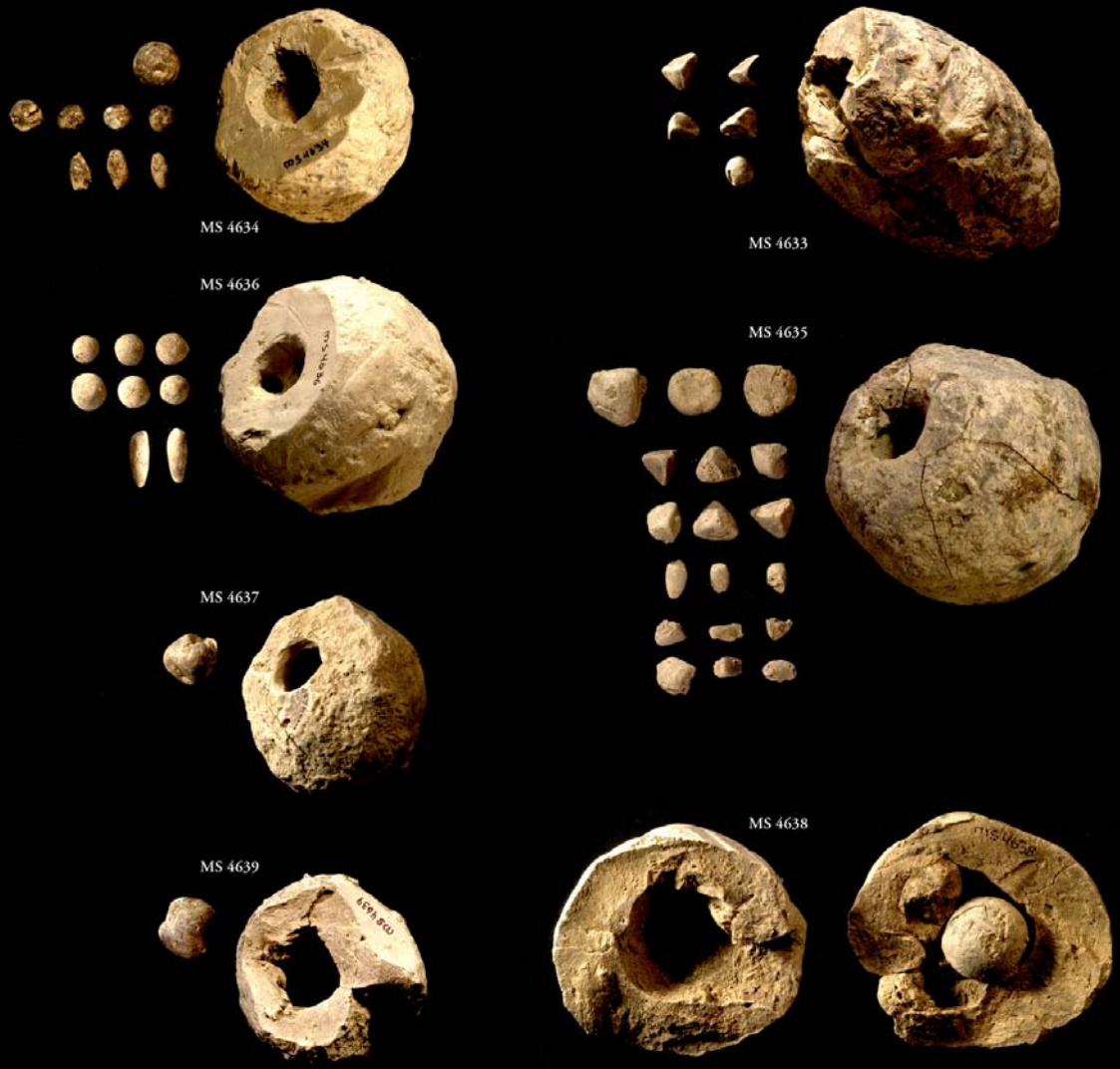
Period	*approx. date (BC)*	*Eanna stratigraphy*
Ubaid	5000-4200	
Early/ Middle Uruk	4200-3800	XIII-IX
Late Uruk	3800-3000	VIII-III
Uruk IVa	3500-3250	IV
Uruk III	3250-3000	III

simple tokens

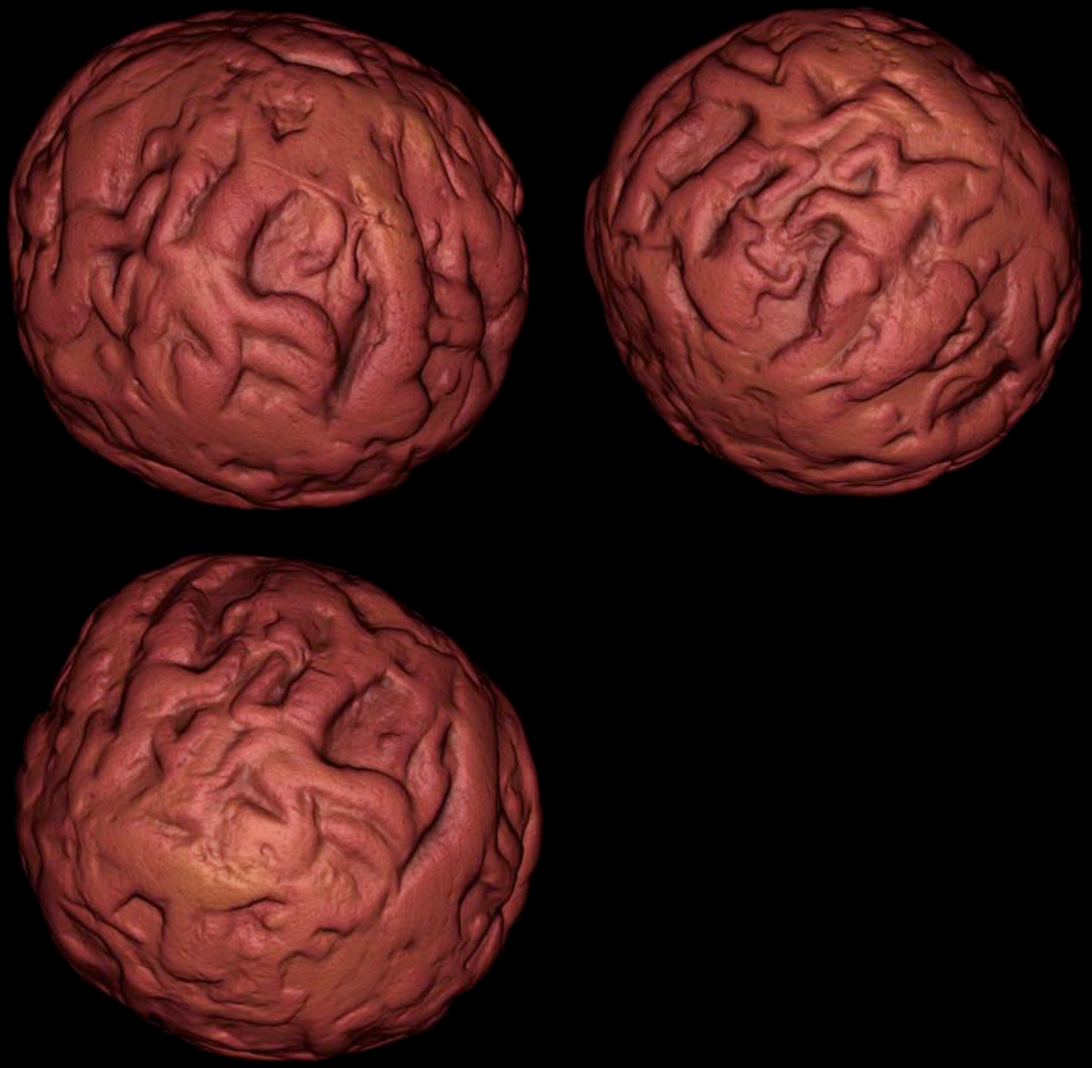
complex tokens; bullae; numerical tablets

proto-cuneiform

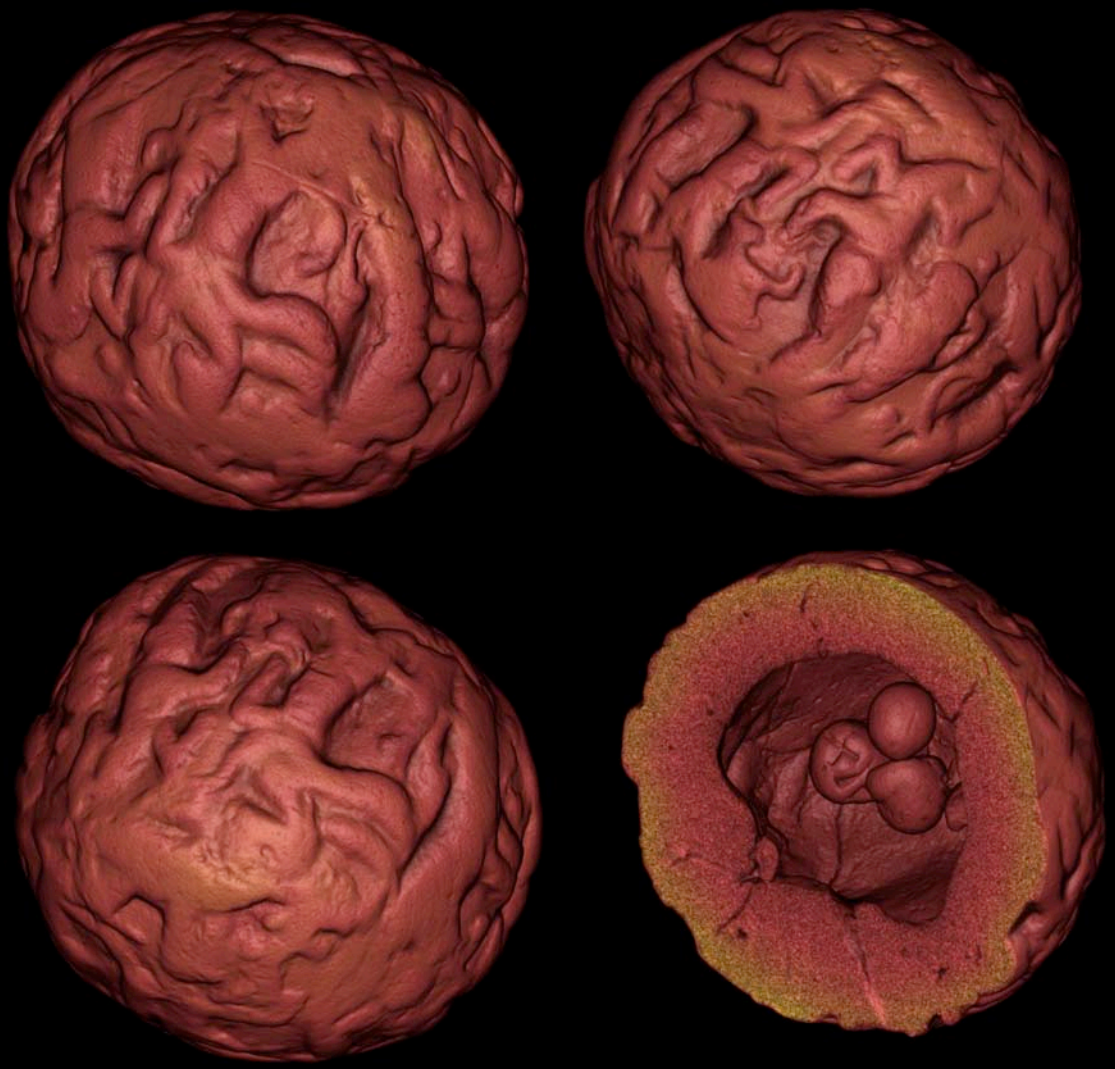
The large majority of clay balls from excavations of 4th millennium sites are, like the examples to the left from the antiquities market (top) and from Susa (bottom), unopened.



Recent acquisitions of the Schøyen collection were cut open to reveal their contents, including one (MS 4631 below) with so-called “silver tokens”



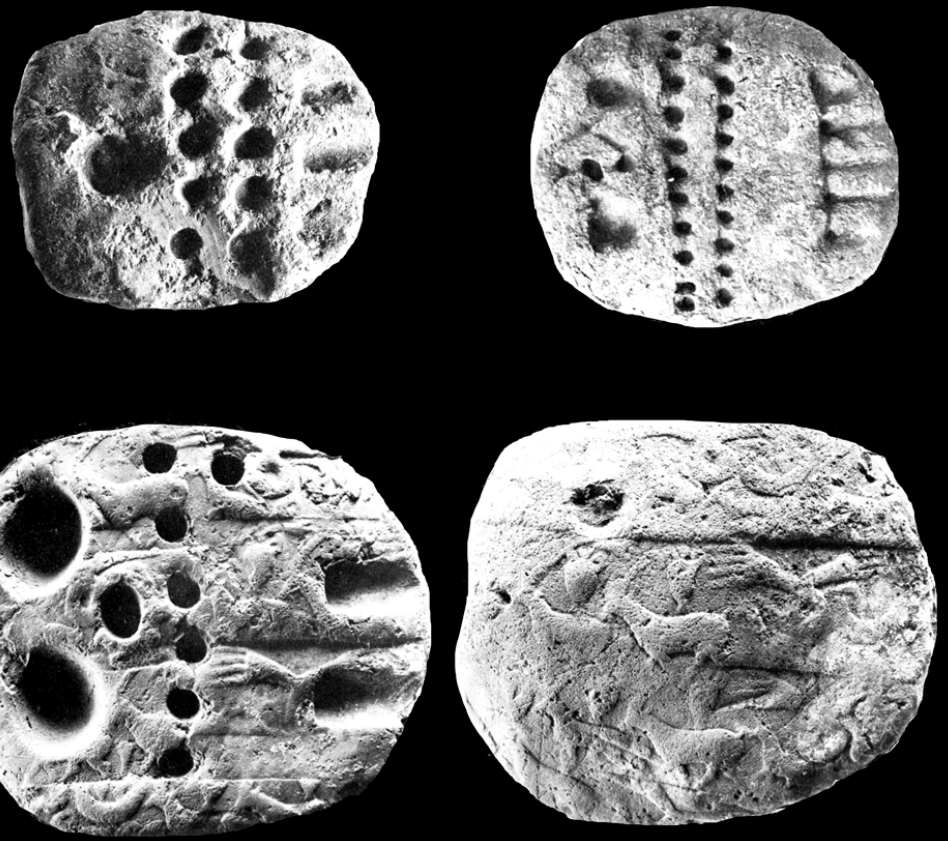
A recent acquisition of
Cornell University



A recent acquisition of
Cornell University

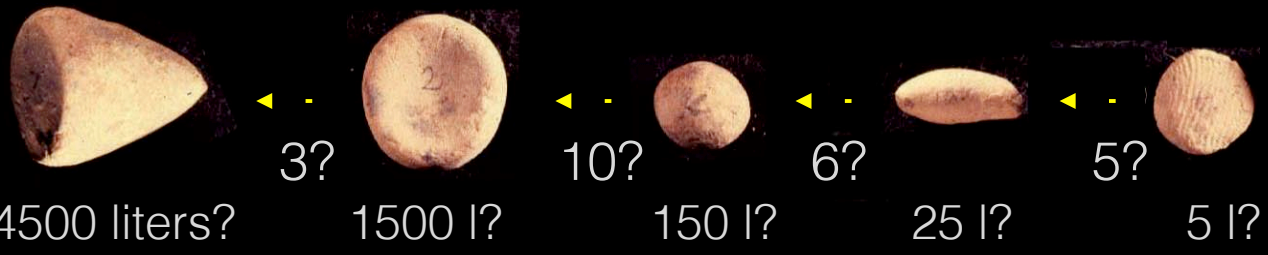
This numerical tablet from Susa, Late Uruk, ca. 3500 BC, contains impressions of an apparent ovoid token (the notation represents ca. 16,000 liters of grain).

The “Wilson series” (Malta)



Numerical tablets
from Jebel Aruda
(ancient Syria)

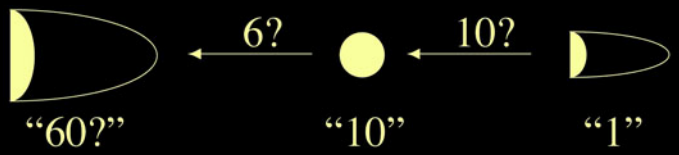
Numerical systems attested in the bullae/tokens:



Grain

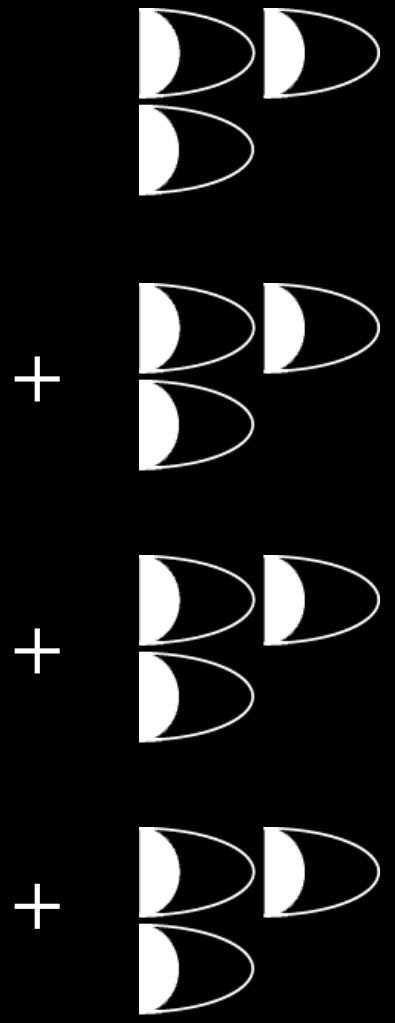
Animals and animal products

Corresponding systems attested in the numerical tablets:



Small account of butter oil delivered in vessels, tallied in the sexagesimal system

W 20676,2 (Uruk)



A small account of grain delivered in baskets
(ca. 3300 BC)

Jürgen Renn

Jöran Friberg

Jens Høyrup

Hans Hunger

Wolfgang Lefèvre

Kurt Vogel

Hans Nissen

Peter Damerow

Determining the archaic number sign systems

COUNT	Folgezeichenvariable						ROW TOTAL
Z9	0.1	1.1	2.1	5.1	17.1	65.1	
1.	152	32	0	134	1	1	320
							55.2
2.	50	9	0	45	0	0	104
							17.9
3.	20	3	0	23	0	0	46
							7.9
4.	25	2	0	13	1	0	41
							7.1
5.	16	1	0	11	1	0	29
							5.0
6.	7	2	1	6	0	0	16
							2.8
7.	5	0	0	8	0	0	13
							2.2
8.	3	2	0	2	0	0	7
							1.2
9.	2	0	0	2	0	0	4
							0.7
COLUMN TOTAL	280	51	1	244	3	1	580
	48.3	8.8	0.2	42.1	0.5	0.2	100.0

Zeichen-wiederholungs-variable

Signs found to follow N_{34} in Uruk notations

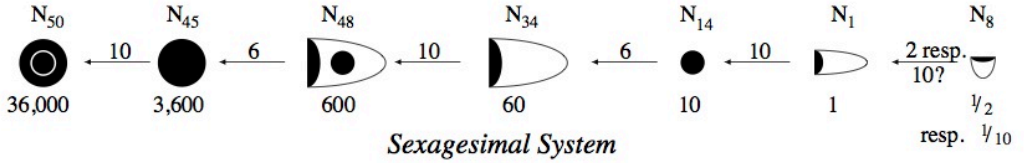
sign	x found	cp. N_1 etc.:
$1N_5$	49	3625x
$2N_5$	39	2179
$3N_5$	30	1400
$4N_5$	28	1072
$5N_5$	22	1065
$6N_5$	0	577
$7N_5$	0	373
$8N_5$	0	359
$9N_5$	0	255
$10N_5$	0	1 (*UET* 2)

$1N_{20}$	47	1941x
$2N_{20}$	31	1130
$3N_{20}$	20	720
$6N_{20}$	7	163
$9N_{20}$	4	72
$10N_{20}$	0	2 (Uruk V)
…		

$1N_{47}$	12	300
$2N_{47}$	8	156
$3N_{47}$	0	13
$4N_{47}$	0	8
…		

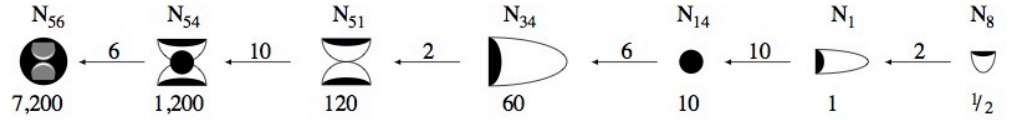
Determining the archaic number sign systems

System used to count most discrete objects, for example, humans and animals, dairy and textile products, fish, wooden and stone implements, and containers



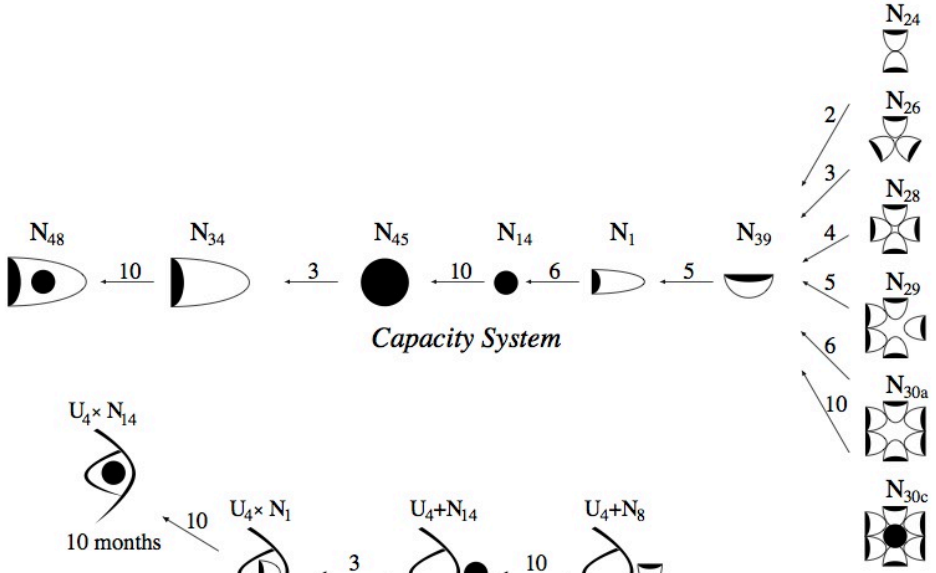
Sexagesimal System

System used to count discrete grain products, cheese, and fresh fish; all objects noted with this system appear to belong to a rationing system



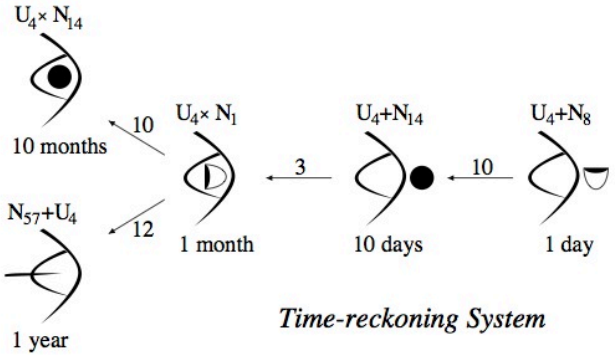
Bisexagesimal System

System used to note capacity measures of grain, in particular barley; the small units also used to designate bisexagesimally counted cereal products

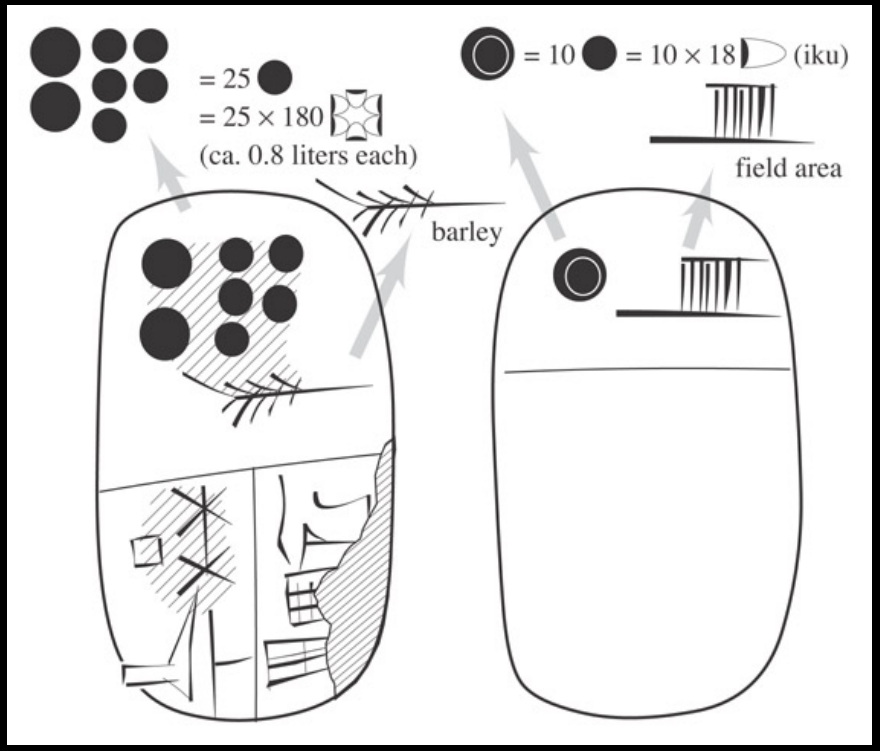


Capacity System

System used to note time and calendar units (twelve 30-day months to a year)



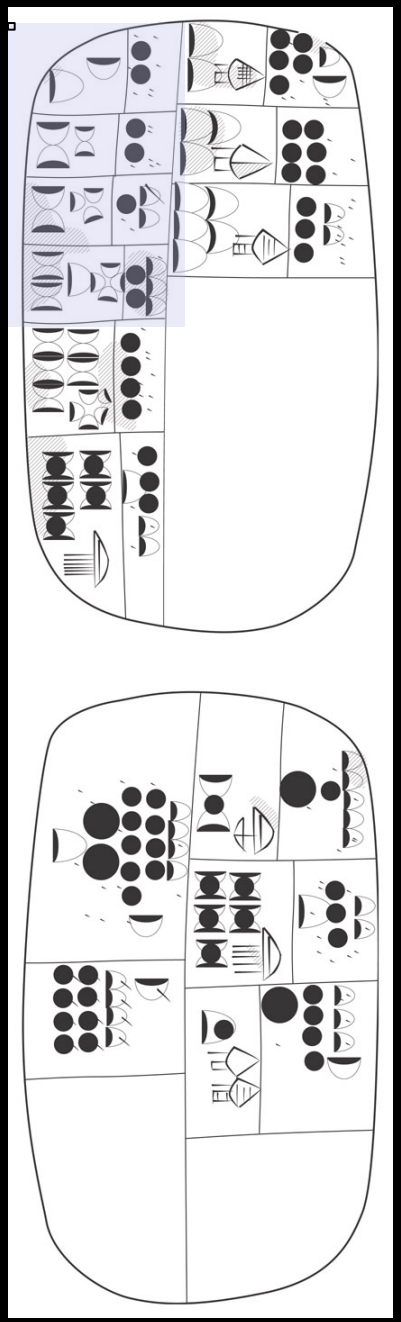
Time-reckoning System

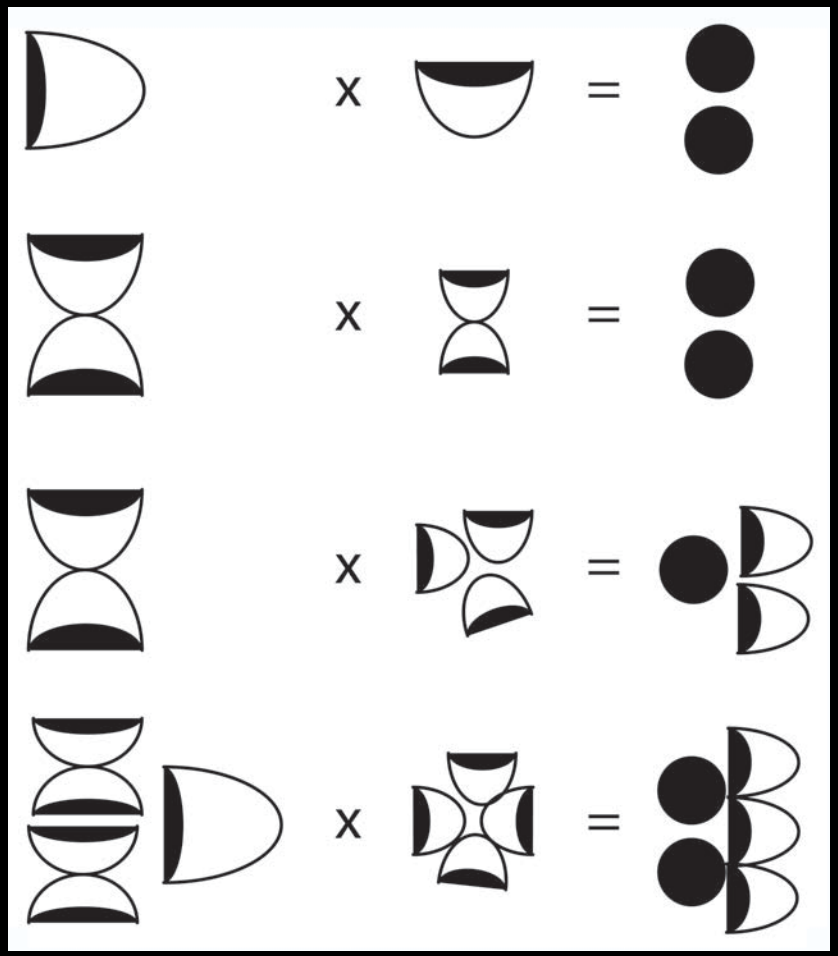


ca. 3600l (150 x 24l) ÷180 iku = 20l/iku

Ur III standard sowing rate: 13 $^{1}/_{3}$ – 20 liters/iku

Text 14: *MSVO* 1, 10 (Jemdet Nasr)





count of discrete units

grain content of each unit

total measure of grain

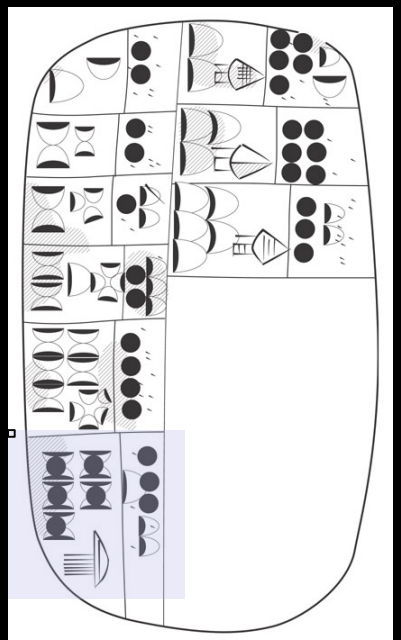
$60 \cdot a = 2N_{14} = 12N_1$
$\therefore a = {}^1/_5\ N_1$

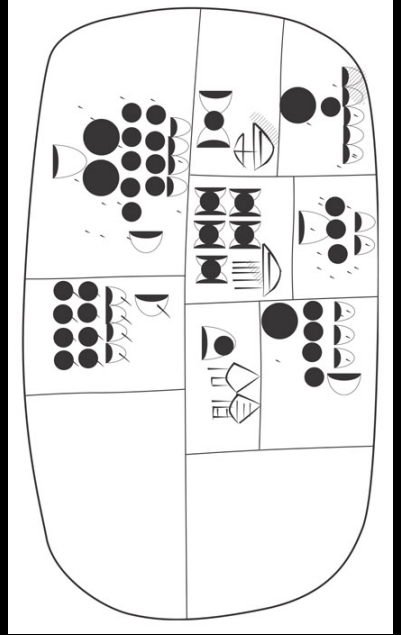
$120 \cdot b = 2N_{14} = 12N_1$
$\therefore b = {}^1/_{10}\ N_1$

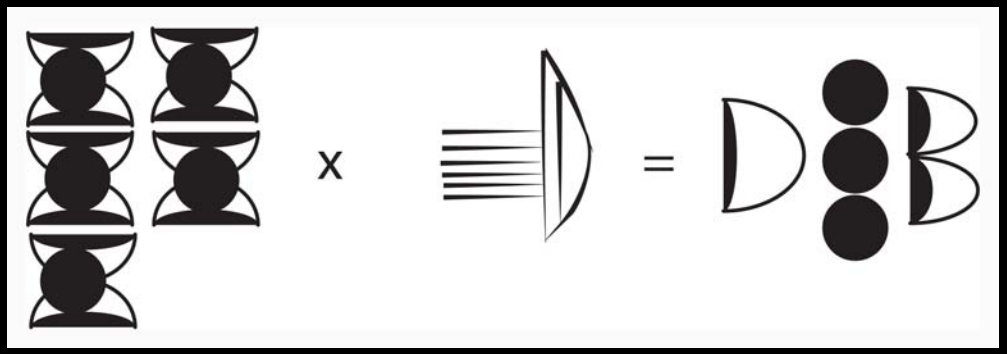
$120 \cdot c = 1N_{14}\ 2N_1 = 8N_1$
$\therefore c = {}^1/_{15}\ N_1$

$300 \cdot d = 2N_{14}\ 3N_1 = 15N_1$
$\therefore d = {}^1/_{20}\ N_1$

MSVO 4, 66 (Larsa ?)





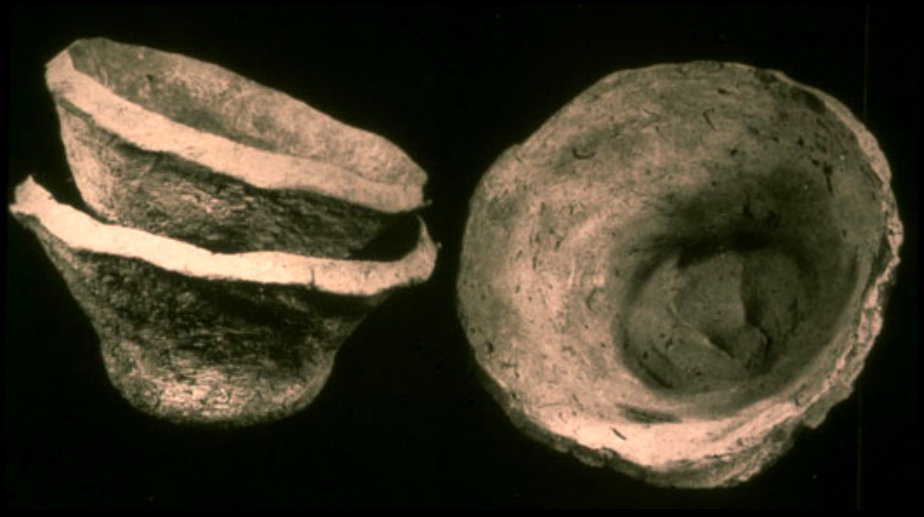


count of discrete units

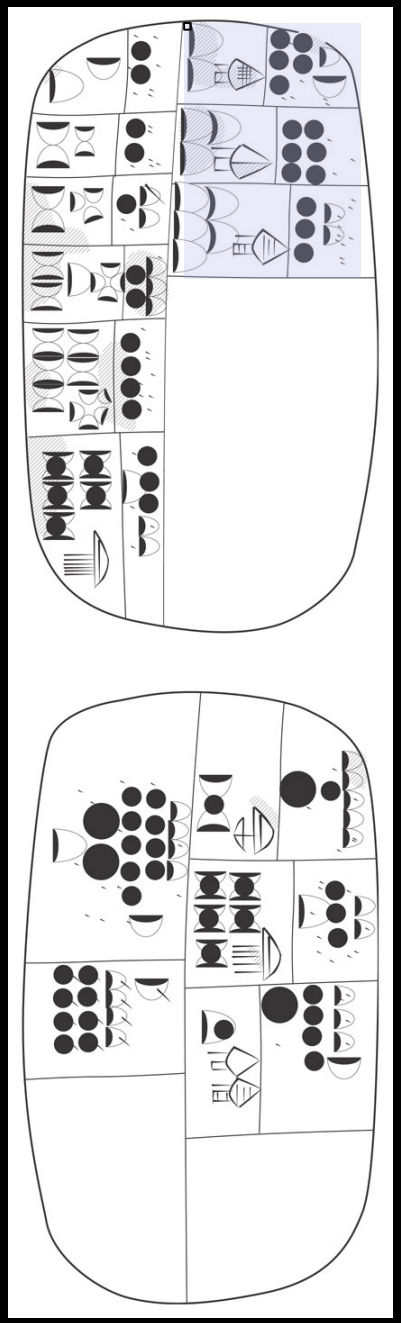
grain content of each unit

total measure of grain

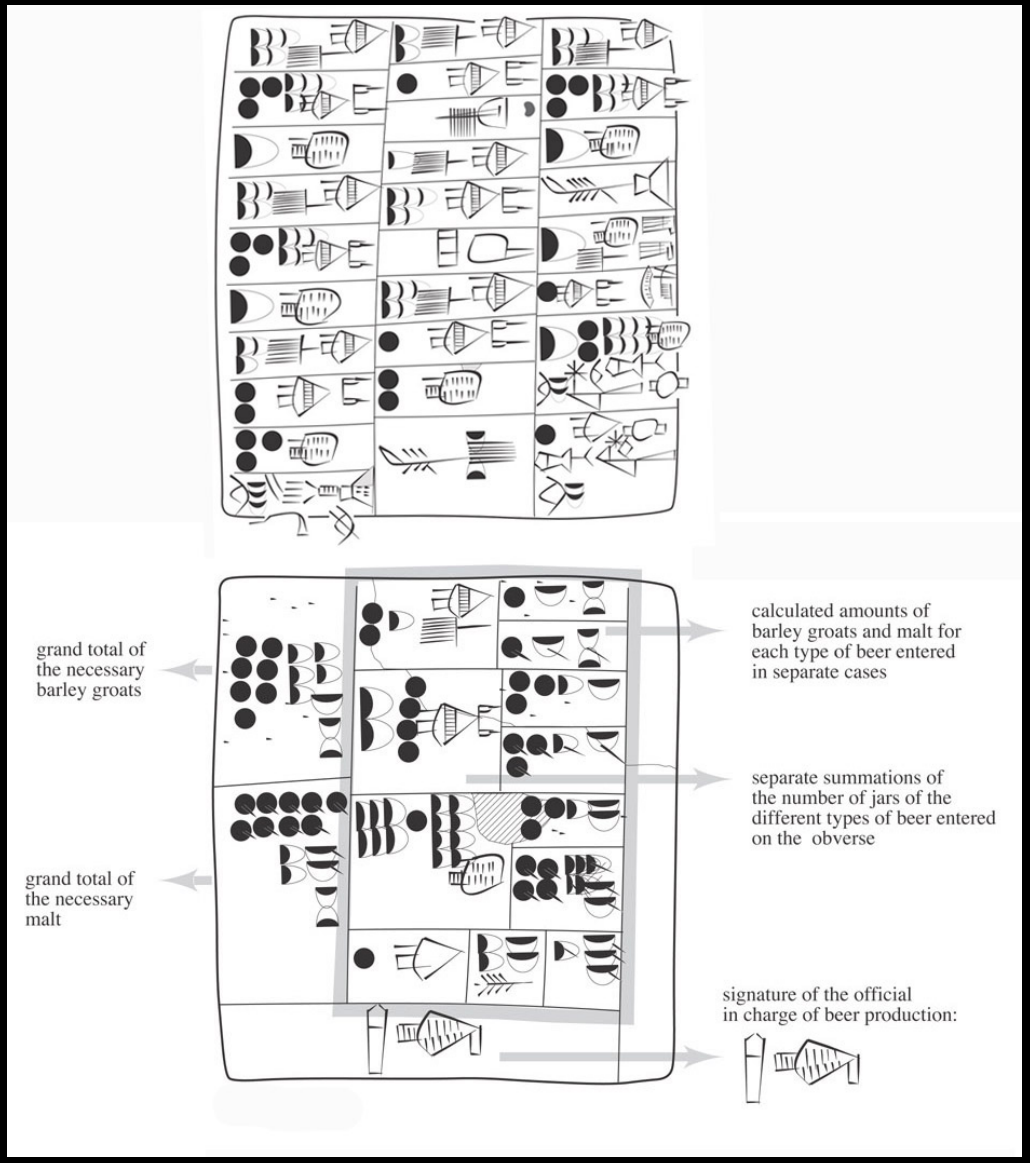
$$6000 \cdot f = 1N_{34}\ 3N_{14}\ 2N_1$$
$$= 200N_1$$
$$\therefore f = {}^1/_{30}\ N_1$$



MSVO 4, 66 (Larsa ?)



MSVO 4, 66 (Larsa ?)



Erlenmeyer 28 was auctioned at Christie's London on 13 December 1988 to Martin Stansfeld, Monaco, for £40,000; on 18 October 2005 to Bolaffi/Rome for £160,000 (with premium: £187,200, ca. €225,000)

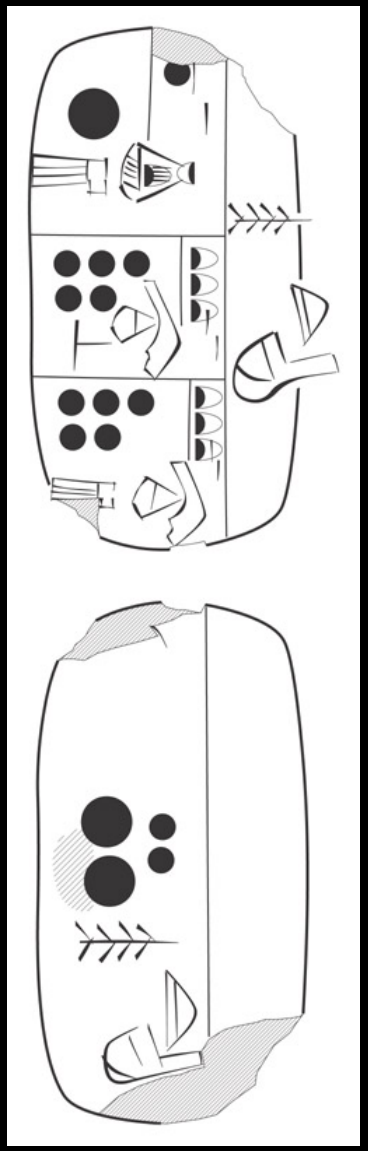
grain expended for $ŠEN_b$ GAL:
$21N_1 \div 63N_{24}$
$= 3N_{24}$ per N_1

for $ŠEN_b$ TUR:
$160N_1 \div 192N_{24}$
$= 1.2N_{24}$ per N_1

for $ŠEN_c$:
$378N_1 \div 194N_{24}$
$\approx .5N_{24}$ per N_1

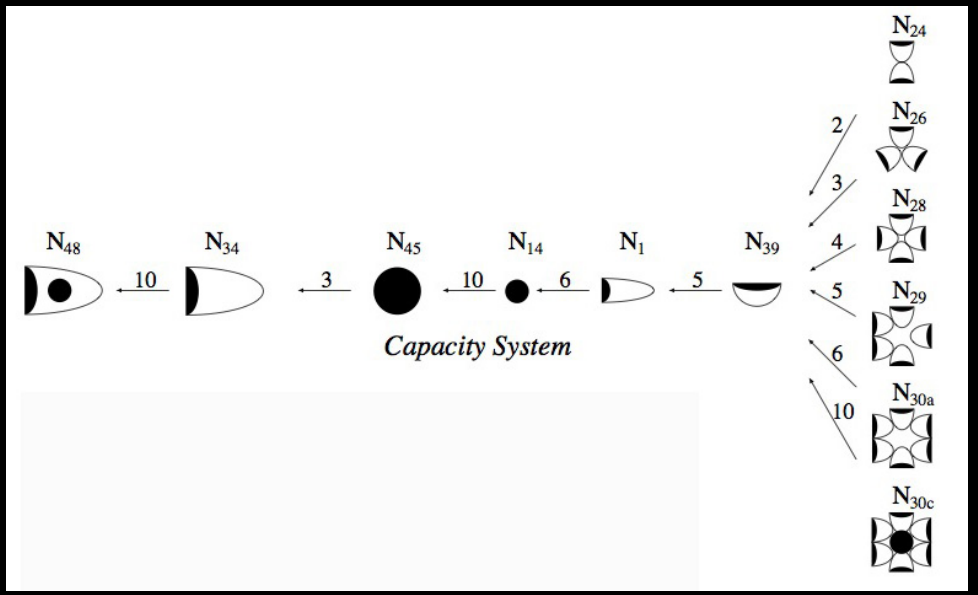
for DUG_a:
$10N_1 \div 24N_{24}$
$\approx .4N_{24}$ per N_1

MSVO 3, 11 (Uruk ?)

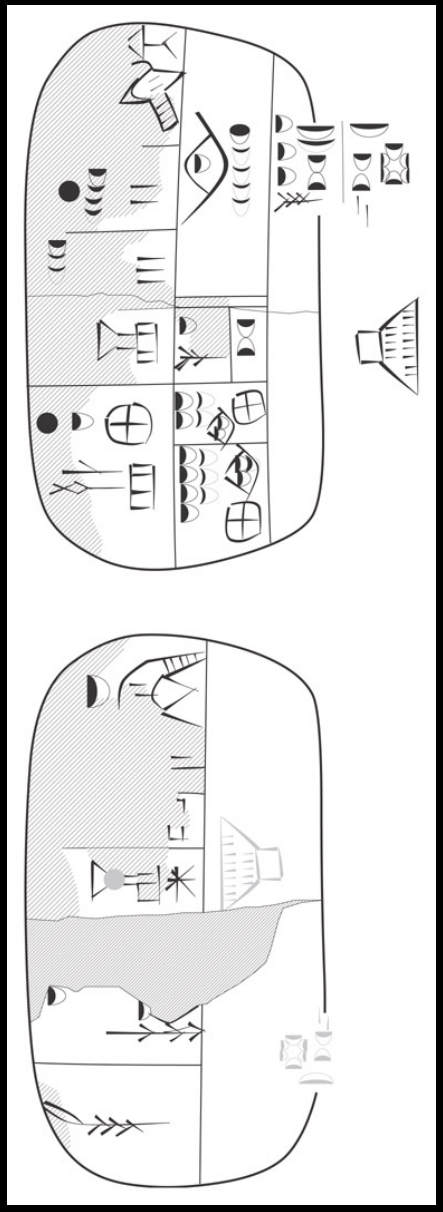


$$1N_{45} \cdot a = 1N_{14}$$
$$\therefore a = {}^{1}/_{10}$$

$$5N_{14} \cdot a = 3N_{1}$$
$$\therefore a = {}^{1}/_{10}$$



MSVO 1, 27 (Jemdet Nasr)



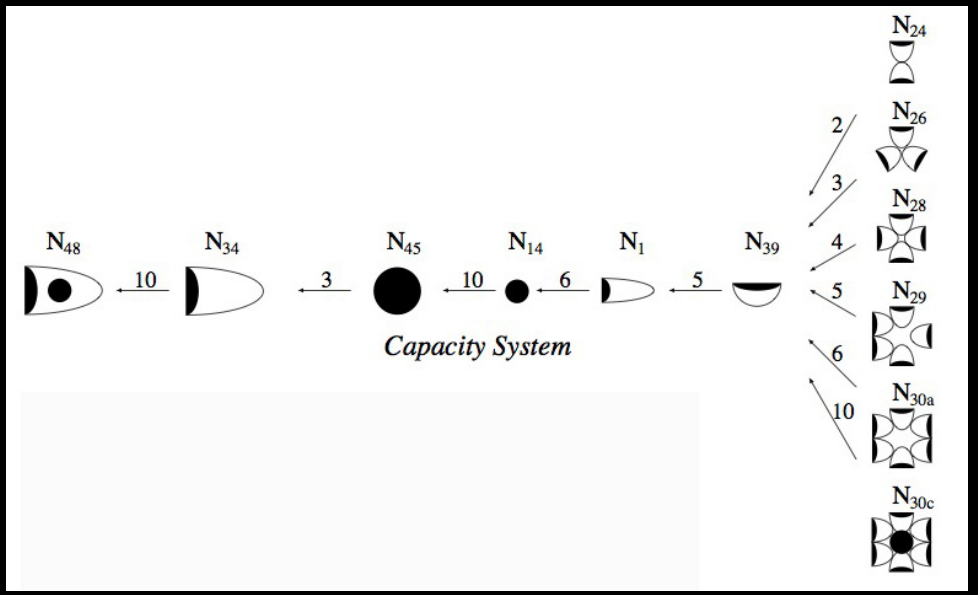
$(3N_1\ 2N_{39}\ 1N_{24}) \cdot a = 1N_{39}\ 1N_{24}\ 1N_{30}$

$\therefore a \approx {}^1/_{10}$

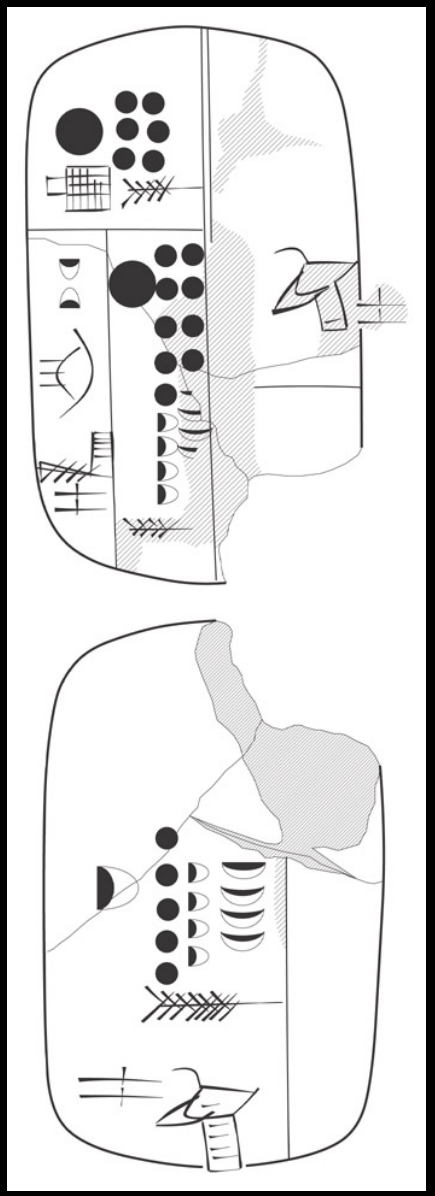
$U_4 x N_1 + 5N_8 = 3N_1\ 2N_{39}\ 1N_{24} = 35N_{24}$

? $U_4 x N_1$ = 1 month of 30 days

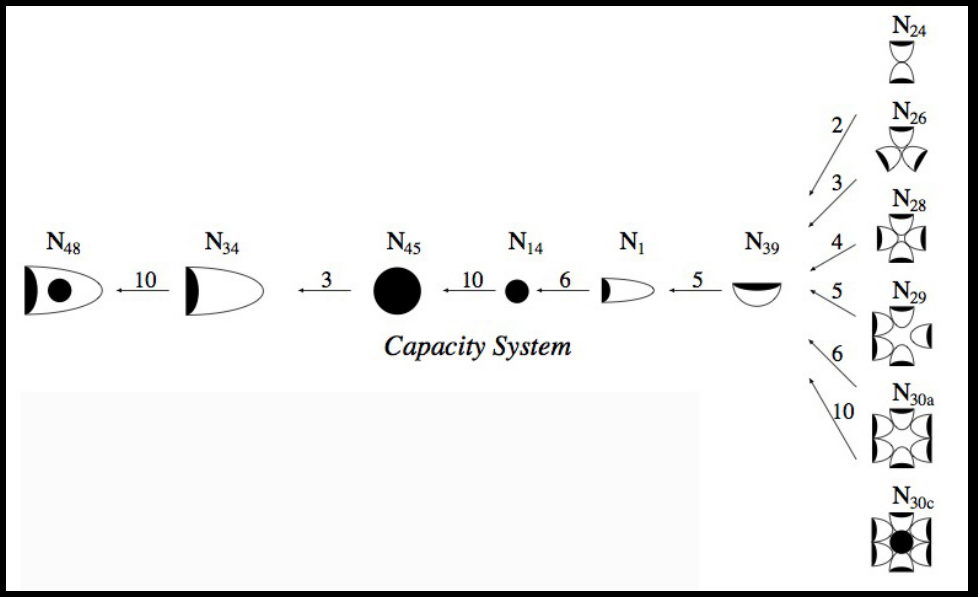
? N_8 = 1 day



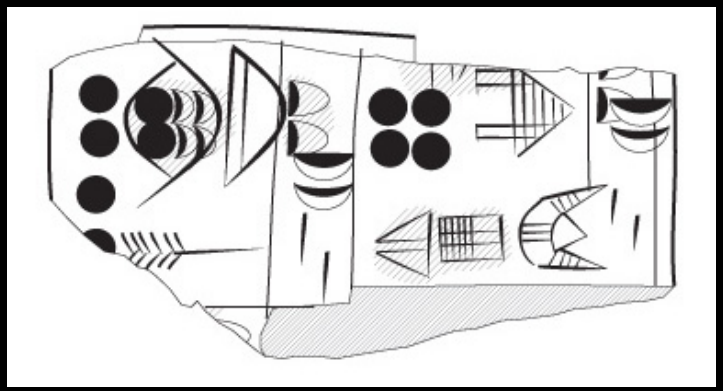
MSVO 1, 121 (Jemdet Nasr)



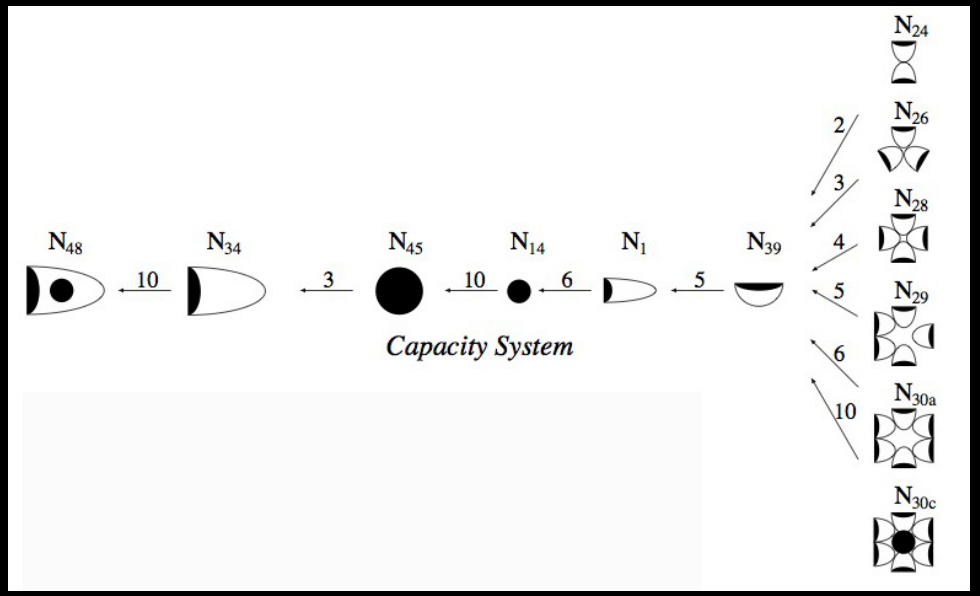
$3N_{57}+U_4 \times 1N_{24} = 1080N_{24}$ $(1N_{57}+U_4 = 360)$
$+ {}^{1}/_{10} = 1188N_{24}$
$= 1N_{45}\ 9N_{14}\ 4N_1\ 4N_{39}$
$(600 + 540 + 40 + 8N_{24})$



MSVO 1, 122 (Jemdet Nasr)



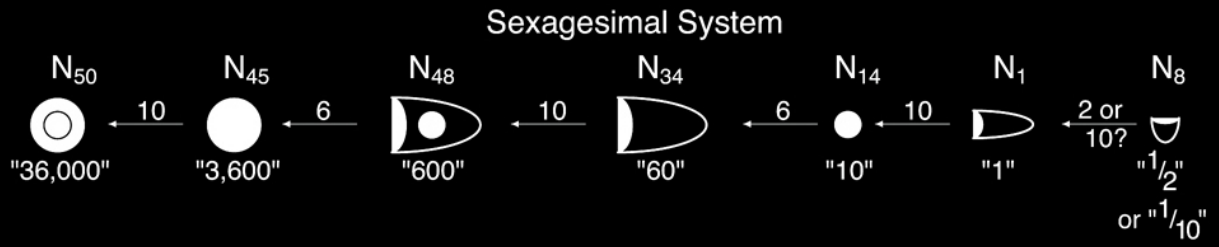
P005429 = MSVO 4, 27
obv. i
1.a. 4(N14)# , |U4x(2(N14).4(N01))| GAR SZE~a
$4N_{14}$ grain in 24 months: GAR(-rations),
$4N_{14} = 720N_{30}$
24 x 30 = 720 days, $1N_{30}$ each day
1.b. 2(N01)# 2(N39~a) , TAR~a
$2N_1$ $2N_{39}$ are "the cut";
$2N_1$ $2N_{39} = 72\ N_{30}$
calculation: 720 ÷ 10 = 72

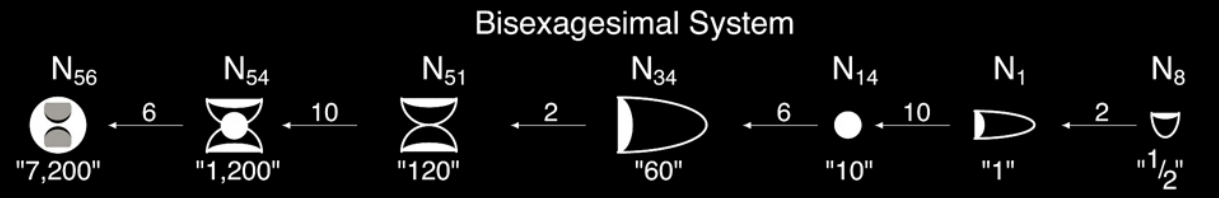


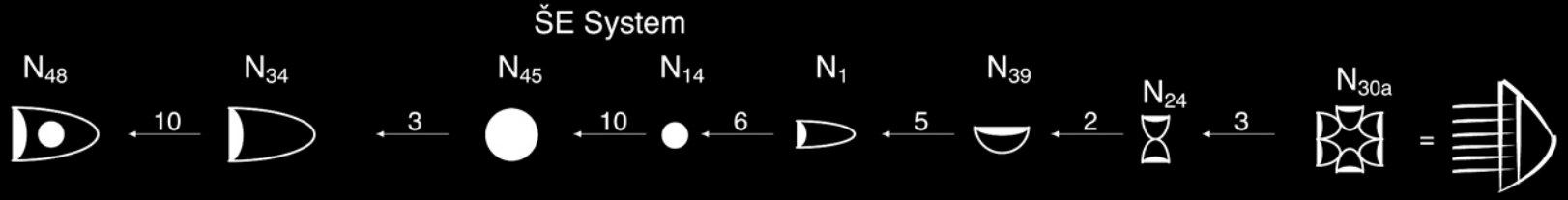
MSVO 4, 27 (Uqair ?)

Vicenzio Formaleoni, *Sources of Errors in the Cosmography and Geography of the Ancients* [*Dei fonti degli errori nella cosmografia e geographia degli Antichi*] (Venice 1789): “The length of the year was therefore indisputably 360 days at the time of the first observers (of the deluge)” [La lunghezza dell’anno era dunque incontrastabilmente de 360 giorni al tempo dei primi contemplatori]

Archaic calendrical system reflected in grain capacity system?







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Archaic calendrical system reflected in grain capacity system?