Conference on the "Historiography of Mathematical Symbolism"

Organized by

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Toni Malet (Institut d'Història de la Ciència, Universitat Autònoma de Barcelona)

September 15-19, 2025
Room 5323, James Clerk Maxwell Building, King's Buildings
Peter Guthrie Tait Road, Edinburgh EH9 3FD

Except for Monday afternoon: **G.32, Murchison House, The University of Edinburgh, 10 Max Born Cres, Edinburgh EH9 3BF**

Argument of the Conference

General histories of mathematics seem to agree with the idea that Vieta should be regarded as the first practitioner to introduce symbolic computations in mathematics. However, this conventional historiography of mathematical symbolism has regularly been challenged. Guglielmo Libri (1803-1869) put forward the thesis that in the 13th century Fibonacci already used similar notations. Franz Woepcke (1826-1864) noted the use of similar types of signs in Diophantos' *Arithmetica* and in Sanskrit works translated into English by Henri Thomas Colebrooke in 1817. Woepcke also reported on his discovery of a 15th-century mathematical work from the Maghreb which he believed testified to the introduction of a form of symbolism into Arabic mathematics. Some decades later, Bibhutibhusan Datta and Avadesh Narayan Singh's *History of Hindu Mathematics* further claimed that Sanskrit works testified to the use of mathematical symbolisms. The same holds true for the historiography of mathematical sources written in Chinese. These documents (and others) have been ever since at the center of discussions dealing with both the actual historical origins of mathematical symbolism and its meaning for mathematics. Furthermore, in a different vein, other points of views on the history of mathematical symbolism have been discussed. Thus, in the 1930s, Otto Neugebauer put forward the thesis that the sumerograms used in cuneiform texts played the part of mathematical symbols, in particular because they did not

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correspond to spoken words. More recently and for a similar reason, Charles Burnett suggested that the decimal place-value notation could be regarded as a form of mathematical symbolism.

The conference will focus on the history of the historiography of mathematical symbolism. The point is not to determine who was actually the first to introduce such notations into mathematics, but rather to analyse what gave rise to these various claims and what historical and philosophical presuppositions about mathematical symbolism underpinned them. Indeed, the claims mentioned above as well as many others illustrate the variety of assumptions about mathematical symbolism that historical analyses have brought into play. It is from this perspective that the conference is interested in the debates to which this issue gave rise.

The seminar series and the conference have two main aims. The first is precisely to explore the historical shaping of the view that mathematical symbolism originated with Vieta. Secondly, the seminar also hopes to examine the properties and the virtues of mathematical symbolism that different actors have foregrounded in their historical analysis. As such, we are interested in different notions of symbol at play in historians' work only in as much as it explains what they understand as symbolism. For example, what features of symbolism were perceived as central when the claim that symbolism was Vieta's invention was (unsuccessfully) challenged by historians on the basis of sources in, e.g., Arabic, Chinese, Latin and Sanskrit? And also, what facets of symbolism have remained overshadowed, or been treated as deriving from properties of symbolism perceived as primary?

Both aims lead to some pivotal questions. A prominent facet of the historical importance given to Vieta's work in relation to mathematical symbolism is the use of literal computation. How did different historians and philosophers understand the specificities and the virtues of this type of computation? How have Vieta's works cast a shadow over most historical discussions on the subject? More largely, what facets of symbolism have been emphasized in relation to the claim that mathematical symbolism was a European invention?

Programme

Monday, September 15, 2025

9:30 am-1pm

Introduction

Karine Chemla (School of mathematics, The University of Edinburgh and SPHERE)
Reopening the History of Mathematical Symbolism Through Remarks on its Historiography
Commentator: Xiaohan Célestin ZHOU

Toni Malet (Institut d'Història de la Ciència, Universitat Autònoma de Barcelona)
John Wallis on Symbolization and the Nature of Viète's "New Algebra".
Commentator: Agathe Keller

2pm-5:30pm—Change of room **for Monday afternoon: G.32, Murchison House, The University of Edinburgh, 10 Max Born Cres, Edinburgh EH9 3BF**

Philip Beeley (Faculty of History, University of Oxford)
After Viète. Symbols and Symbolic Representation in the Mathematical Writings of John Wallis (1616-1703)
Commentator: Prashant Kumar

E. Hunter
Isaac Barrow blowing Archimedes out of proportion: A close look at his commentary for Sphere and Cylinder *Book 2 Proposition 4*
Commentator: Jens Høyrup

Tuesday, September 16, 2025

9:30 am-1pm

Jens Høyrup (Roskilde University, emeritus)
The revised Nesselmann categories applied to European algebraic writings between 1300 and 1650
Commentator: Christophe Eckes

Agathe Keller (SPHERE, CNRS—University Paris Cité)
How Historians and Orientalists have Debated the Existence of Symbolism in Sanskrit Mathematical Texts. A First Overview centered on the Figure of Léon Rodet
Commentator: Philip Beeley

2pm-5:30pm

Zehra Bilgin (Ph.D, Istanbul Medeniyet University, Institute for the History of Science, İstanbul, TÜRKİYE)
The Historiography of Mathematical Symbols in Salih Zeki's Kāmūs-i Riyādiyyāt
Commentator: E. Hunter

Alex Garnick (Harvard University and SPHERE) and **Karine Chemla** (School of mathematics, The University of Edinburgh and SPHERE)
Revisiting Salih Zeki's Thesis about the Origins and History of Arabic Algebraic Notation.
Commentator: Richard Oosterhoff

Wednesday, September 17, 2025

9:30 am-1pm

S. Prashant Kumar (University of Chicago**)**
The Chakravala's New Clothes: De Morgan on Symbolism, Analogy, and Epistemology in Bhāskara's Solutions to Indeterminate Equations
Commentator: Marie-José Durand-Richard

Benjamin Wardhaugh (University of Oxford)
Mathematical symbolism and its history in the Mathematical and Philosophical Dictionary *of Charles Hutton (1795/1815)*
Commentator: Arilès Remaki

Free afternoon

Thursday, September 18, 2025

9:30 am-1pm

Marie-José Durand-Richard (Honorary Lecturer Université Paris 8 Vincennes & Researcher associated to SPHERE, CNRS, CNRS—University Paris Cité)
Historiography of Mathematical Notations by Cambridge's Algebraists (1820-1845)
Commentator: Isobel Falconer

Ivahn Smadja (Nantes Université, CAPHI - Institut Universitaire de France (IUF))
Signs, symbols and operations: Humboldt on numeral systems and the historiography of mathematical symbolism
Commentator: Toni Malet

2pm-5:30pm

Christophe Eckes (Archives Henri-Poincaré, université de Lorraine)
Bourbaki's historiography on algebraic notations and symbolism
Commentator: J.P. Ascher

David Waszek (post-doctoral fellow, Ecole Normale Supérieure)
Uncontrolled symbolic computations in the history of the differential calculus and in non-Western mathematics: A first exploration of late 19th-century views
Commentator: Ksenia Tatarchenko

Friday, September 19, 2025

9:30 am-1pm

Arilès Remaki (Johannes Gutenberg Universität Mainz)
Exponentiation: a notational issue?
Commentator: Karine Chemla

Zhou Xiaohan (Institute for the History of Natural Sciences, Chinese Academy of Sciences, Beijing)
Differences in the Understanding of Mathematical Symbolism Between Mathematicians and Historians of Mathematics in 19th and 20th century China

Commentator: Zehra Bilgin

2pm-5:30pm

Ksenia Tatarchenko (JHU, Medicine, Science, and the Humanities program)
Reddening Signs: What is Marxist about the Russian Language Historiography of Mathematical Symbolism?
Commentator: Ivahn Smadja

General discussion, launched by Alex Garnick and David Waszek

************************************ **Abstracts**

Philip Beeley (Faculty of History, University of Oxford)
After Viète. Symbols and Symbolic Representation in the Mathematical Writings of John Wallis (1616-1703)

Abstract: As is well known, the long-serving Savilian professor of geometry at Oxford John Wallis devoted a considerable amount of space in his published and unpublished writings to questions of algebraic notation and symbolic representation, focusing particularly on the work of Viète, Harriot, and Oughtred. In this paper I set out and discuss some of Wallis's key mathematical arguments and epistemic considerations, but including also the need to take account of what could reasonably be carried out in the printer's workshop.

Zehra Bilgin (Ph.D, Istanbul Medeniyet University, Institute for the History of Science, İstanbul, TÜRKİYE)
The Historiography of Mathematical Symbols in Salih Zeki's Kāmūs-i Riyādiyyāt

Abstract: Salih Zeki (1864–1921), an Ottoman mathematician and historian of science, authored the pioneering mathematical encyclopedia *Kāmūs-i Riyādiyyāt*. The first volume, published in 1897, marked the inception of this ambitious 12-volume project; however, only the initial volume was released, with the second prepared but unpublished, and subsequent volumes remaining in manuscript form. This study investigates Salih Zeki's conceptualization of mathematical symbolism and notation, focusing on his articles "Numbers" (*A'dād*) and "Signs" (*Ishārāt*). In his taxonomy, Zeki differentiates between operational symbols (*ishārāt -ı ʿamaliyya*) that represent arithmetic and algebraic operations and relational symbols (*ishārāt -ı nisbiyya*) that express relationships such as equality and inequality. His historical narrative strongly emphasizes the role of operational symbols in facilitating mental reasoning and computational efficiency, reflecting a pragmatic understanding of mathematical practice.
Although Zeki briefly acknowledges François Viète's groundbreaking approach to symbolizing both known and unknown quantities with letters, he does not deeply engage with Viète's broader project of algebraic abstraction. Instead, Zeki's historiographical focus remains rooted in the evolution of operational signs. This suggests that he valued the practical applicability of symbolic notation over abstract structural innovation.
By analyzing Zeki's selective historiographical choices, this study contributes to a deeper understanding of how mathematical knowledge, especially in its symbolic dimensions, was adapted, reframed, and transmitted within the intellectual context

of the late Ottoman Empire.
Such an inquiry not only enriches the historiography of mathematics but also illuminates the broader intellectual transformations occurring in the late Ottoman world.

Karine Chemla (School of mathematics, The University of Edinburgh and SPHERE)
Reopening the History of Mathematical Symbolism Through Remarks on its Historiography.
Abstract: Since the beginning of the nineteenth century, in some historiographies the history of mathematical symbolism, just as the history of mathematical proof, has been one of the key themes to assert the distinctiveness of Europe and to set Europe against the rest of the world (or certain parts of Europe against others). In the nineteenth and twentieth centuries, much historical work has been devoted to mathematical sources produced outside Europe since Antiquity. In this context, comparison between non-European and European sources has played a key part. This remark holds true in particular for the historiography of mathematical symbolism, which has been largely informed by the developments in mathematical symbolism during the early modern period. This point can be illustrated by Franz Woepcke's (1826-1864) article from 1854. Woepcke's search for notations similar to those of Diophantos and Vieta led him to discover a new Arabic manuscript, which became the first known Arabic work containing such notations. Moreover, Woepcke's analysis of this manuscript in 1854 focused on the notation of the same mathematical objects for which Diophantos and Vieta had used specific notations, that is, equations and polynomials. In this sense, Woepcke's search was retrospective in two ways. More generally, historical research into mathematical symbolism has become almost synonymous with research into algebraic notation. This is again illustrated almost a century later by Adolf Pavlovitch Youschkevitsch's (1906-1993) remarks on symbolism in ancient Chinese sources, which nevertheless attests to a twist. Indeed, Youschkevitch noted that ancient Chinese mathematical sources testify to a certain form of symbolism for the notation of polynomials and equations, even though the nature of this symbolism differs from that used by Diophantos and Vieta. Woepcke's and Youschkevitch's retrospective focus on *algebraic* notation led them to severe the notations they were interested in from cognate notations in the same sources, notably, the decimal place-value notation and that of operations performed using it. More recently, in 2002, Charles Burnett's work on precisely the decimal place-value notation has introduced a different approach to the history of mathematical symbolism. Indeed, Burnett has noted that in Sanskrit, Arabic, and later Latin sources, this numeration system—just as algebraic symbolism—was not representing spoken language. He related this feature to the fact that both types of notation (place-value notation and algebraic notation) had circulated across language boundaries. Incidentally, this remark establishes a relation between two types of notation that both Woepcke and Youschkevitch had treated as unrelated to each other. Interestingly, in 1932-1933, but in a different way, Otto Neugebauer (1899-1990) also suggested that the separation between the written signs and the oral language was a feature that enabled historians to identify a form of mathematical symbolism in cuneiform sources. As these authors highlight, what makes important the historiography on mathematical symbolism is that it sheds light on the diversity of interpretative frames of what must be taken to be a proper mathematical symbolism. While all these approaches remain informed by the

experience of modern symbolism, they do so from different perspectives, with each starting from a different feature. These works invite us to reconsider how we might approach the history of mathematical symbolism anew.

Marie-José Durand-Richard (Honorary Lecturer Université Paris 8 Vincennes & Researcher associated to SPHERE, CNRS, CNRS—University Paris Cité)

Historiography of Mathematical Notations by Cambridge's Algebraists (1820-1845)

Abstract: In the first half of the 19th century, Charles Babbage (1791-1871) and George Peacock (1791-1858), as members of the transient "Analytical Society", were concerned with the issue of algebraic notations. Introducing the Leibnizian notation of the differential calculus at Cambridge University, they conducted a broader reflection on the inventive processes which organized their development. In the 1820s, Babbage wrote several papers, such as an unfinished *Philosophy of Analysis*, in which he focused on the principles governing the invention of notations for an efficient language of signs. In 1826, Peacock wrote a huge "History of Arithmetic", published in the *Encyclopaedia Metropolitana* in 1845. This large inquiry into the vocabulary that all known tribes and nations used for elementary computations was seeking for a universal genesis from arithmetic to algebra, founded on traces of the decimal system and on operations. Both of them were mainly guided by John Locke's empiricist philosophy in his *Essay on Human Understanding* (1690) (for which experience, through ideas and words, was organized by the operations of the mind), and by the debates of this period between Indologists and scholarly travelers, in France and in Great-Britain.

Christophe Eckes (Archives Henri-Poincaré, université de Lorraine)

Bourbaki's historiography on algebraic notations and symbolism

Abstract: Bourbaki's historical notes are closely linked to the genesis of the *Éléments de mathématique*, a long-term collective editorial undertaking that began in the second half of the 1930s. These historical notes were initially appended separately in various fascicles of the *Éléments* published between the 1940s and the 1960s. These notes were then brought together in a single volume entitled *Éléments d'histoire des mathématiques*, the first edition of which dating back to 1960. The historiographical choices to which these notes bear witness can therefore only be properly understood if we take into account the disciplines, the mathematical theories and the conceptual framework that Bourbaki members prioritized with the publication of this collective work, as well as the objectives they hoped to achieve through this editorial undertaking - among which we can mention the desire to access to a dominant position in the field of mathematics, the project to profoundly transform university-level mathematics education, and the aim of carrying out a terminological and conceptual reform covering entire areas of pure mathematics.

A substantial analysis of Bourbaki's historical notes was provided by mathematics historians David Aubin and Anne-Sandrine Paumier in 2016. Their aim was to examine the role of collective mathematical practices in the accounts offered by Bourbaki members in their historical notes. In this talk, I would like to take a fresh look at Bourbaki's historical notes, particularly those from the algebra book, by addressing the following questions. (i) What secondary sources did Bourbaki members draw on to write the passages that deal with "algebraic notation," "literal

symbols," or what Bourbaki members call "abstract algebraic calculation"? In particular, we will show that Bourbaki members made numerous borrowings from the French translation of Hermann Schubert's article on the fundamental principles of arithmetic for the *Encyklopädie der mathematischen Wissenschaften*. They also referred to the first volume of Johannes Tropfke's *Geschichte der Elementar-Mathematik* and to Henri Bosmans' work on Pedro Nuñez's *Libro de algebra*. (ii) What are the different temporal and geographical scales adopted by Bourbaki members to develop their historical account of algebraic notations and operations? (iii) Which historiographical, conceptual, thematic, or theoretical categories did Bourbaki use to address this question? We will see, on the one hand, that a substantial part of Bourbaki's historical account is based on an opposition between geometric representations and algebraic notations. On the other hand, we will emphasize that their narrative on algebraic notations and operations is dominated by a retrospective illusion centered on the notion of composition laws, which is symptomatic of a conception of the history of mathematics oriented toward the emergence of structural mathematics, of which the *Éléments de mathématiques* would be the quintessence.

Alex Garnick (Harvard University and SPHERE) and **Karine Chemla** (School of mathematics, The University of Edinburgh and SPHERE)

Revisiting Salih Zeki's Thesis about the Origins and History of Arabic Algebraic Notation..

Abstract: In this presentation, we aim to recover the work of the late-Ottoman historian of mathematics, Salih Zeki Bey (1864-1921), in particular, the thesis about the history of Arabic algebraic notation that he advanced within the pages of the *Journal asiatique* in 1898. Responding directly to the upswell of orientalist scholarship on the subject—principally, the work of Nesselmann, Woepcke, and Rodet—Salih Zeki introduced to the literature a "complete" Arabic algebraic notation which he came across, during the course of his research, in the margins of a fifteenth-century *mashreqi* manuscript at the Süleymaniye Library in Istanbul. Salih Zeki's study of this manuscript and its notation eventually led him to make some striking claims about the origins and necessity of "abbreviating signs" in any algebraic practice, that is, their tacit use by actors whose works would otherwise seem not to contain any traces of a notation, most saliently for him being al-Khwārizmī. We will highlight key features of Salih Zeki's approach to the history of Arabic algebraic notation, in particular his understanding of the meaning of algebra and of the related use of algebraic notation, as well as his historiography of signs and of algebraic notation, notably the issue of the relation between written notation and its "pronunciation." In addition to expounding his thesis and way of reading the sources, we aim to read *beyond* Salih Zeki, so to speak, bringing out the connections between algebraic notations and other kinds of mathematical symbolism (e.g., the decimal place-value notation) that were evident in the sources themselves but that nonetheless escaped the notice of this nineteenth-century historiography, Salih Zeki included.

Jens Høyrup (Roskilde University, emeritus)

The revised Nesselmann categories applied to European algebraic writings between 1300 and 1650

Abstract: At the seminar, I discussed Nesselmann's *Stufen* and the modifications of his scheme necessitated by a situation where a large number of sources to which he did not have access. At the present occasion I shall discuss the use of signs – some as symbols in Nesselmann's sense, some definitely not – in abbacus writings, in *Rechemneister* algebra (*Coß*), and by various French writers.

E. Hunter (University of Chicago)

Isaac Barrow blowing Archimedes out of proportion: A close look at his commentary for Sphere and Cylinder *Book 2 Proposition 4*

Abstract: Archimedes' *Sphere and Cylinder* II contains a series of geometric problems involving spheres and spherical segments defined by specific ratios. Among these, Proposition 4 – which divides a sphere into segments whose volumes are in a given ratio - has garnered the most attention. This is due partly to Archimedes reducing the problem to what we recognize as a cubic equation, and partly because the manuscripts lack the promised analytical exposition and constructive synthesis of this auxiliary problem. In 1675, Isaac Barrow's edition of Archimedes is published in which he translates the ancient Greek mathematical discourse into symbolic expression. For Proposition 4 (numbered 5 in his edition), Barrow interrupts his proof to voice frank disbelief that the Greek geometer could have reached such sophisticated results within the cumbersome ancient Greek theory of proportionality. What is missing, in Barrow's view, is any clear articulation of method. Barrow argues that if Archimedes truly derived the solution himself, it must have been "by chance rather than by reason," and that consistent success through such means is "hardly conceivable." Next, Barrow himself does not attempt to reconstruct the gap. Instead, he cites Eutocius' "laborious and lengthy" reconstruction and strongly advises readers either to consult Huygens' geometric demonstration - likely developed via algebraic means according to Henk Bos[2] **-** or to derive the solution independently using Descartes' general algebraic methods. Barrow thus implicitly rejects not the validity of Archimedes' results but rather the methodological feasibility of pure proportional reasoning, dependent on diagram-specific manipulations and ad hoc permutations. This presentation examines Barrow's incredulity and discomfort as indicative of the degree to which symbolic algebra had become fundamental to mathematical reasoning in his era, effectively setting an epistemic benchmark against which ancient mathematical practices were judged. By comparing the proofs, the presentation elucidates the virtues ascribed to symbolic algebra in contrast to those attributed to ancient Greek mathematical methods. This reveal how symbolic formalism was not merely a new technique but a new criterion for mathematical intelligibility, against which ancient Greek mathematics was found lacking.

Agathe Keller (SPHERE, CNRS—University Paris Cité)

How historians and orientalists have debated the existence of symbolism in Sanskrit mathematical texts. A first overview centered on the figure of Léon Rodet.

Abstract: When Henry Thomas Colebrooke (1765-1837) published in 1817 translations into English of seventh and twelfth century mathematical texts in

[2] *Redefining Geometrical Exactness* (2001).

Sanskrit, a substantive part of his introduction was devoted to establishing that these texts “possessed” an algebra. Until today, this remains debated. By the end of the nineteenth century, partly due to the works of Georg Heinrich Ferdinand Nesselmann (1811-1881), much of the discussion had to do with establishing whether or not when solving problems in algebra Sanskrit mathematical texts used symbols. However, these early historians of South Asian mathematical texts also noticed all sorts of other notations and marks that could be employed when practicing mathematics. In particular, part of the literature was also devoted to the use or not of a place-value notation when dealing with arithmetical computations. This talk is a first exploration of the changing definitions and perceptions of the importance of symbols and notations in the discussions on Sanskrit mathematical texts from the early questions of the end of the 18th century raised by John Playfair (1748-1819) to the arguments used by Johan Frederik “Frits” Staal (1930-2012) and Albrecht Heeffer in the second half of the 20th century and the beginning of the 21st. In particular I would like to describe how notations were or not ascribed or related to language. In this first essay I will focus on the different positions that Leon Rodet (d. 1895) took on these questions, and the influence he had on the subsequent historiographies. We will see how this author puts the emphasis on the non-discursive practices of past mathematicians.

S. Prashant Kumar

The Chakravala's New Clothes: De Morgan on Symbolism, Analogy, and Epistemology in Bhāskara's Solutions to Indeterminate Equations

Abstract: In this paper I approach questions surrounding the conflation of concept and notation by looking at European commentaries regarding the presence or absence of continued fractions in Sanskrit mathematics. I examine the case of second order Diophantine equations, where two methods of solution, Lagrange's continued fractions and Bhāskara's *chakravala,* solve the same problems, but cannot be placed in line-by-line correspondence. Edward Strachey, working with a Mughal-era Persian translation in 1813, argued that Bhāskara's method was essentially the same as Lagrange's method. H.T. Colebrooke, working with Sanskrit texts a few years later, goes further and claims that Bhāskara anticipated 17th century European methods, but stops short of attributing continued fractions to Bhāskara. But a generation later, in the 1830s, the famously cantankerous Augustus De Morgan reviewed Colebrooke on the system of notation deployed by Sanskrit algebraists. Reviewing Strachey and Colebrooke's discussion of Bhāskara in the *Penny Cyclopedia*, his reading of their work highlights the non-transmission of symbolism (which he understands as both stand-ins for unknown quantities and for the “steps or operations of an algebraic process”) from Sanskrit into Arabic, which for him suggests the possibility of another route for the westward spread of algebraic notation. For De Morgan, the analogy between the *kuṭṭaka* and continued fractions hinges on the difference between 'rule' and 'notation': "The Hindus give no use of continued fractions except [the *kuṭṭaka* algorithm], though it is obvious, from the skill with which they manage the reduction of fractions into nearly equal fractions of more simple terms, that they must have applied continued fractions, directly or indirectly, probably by means of this very rule.” De Morgan clarified that he "does not mean to say that [Bhāskara] had continued fractions, but only the processes involved in

the use of them, and the power of attaining their results." I end by attempting to answer the following questions. Does conceptual similarity *supervene* on notation, and why or why not? How, for De Morgan, do ideas of notation and concept link to ideas of symbolism or formalism? And, moreover, what do these historiographical positions tell us about the changing stakes involved in the attribution of civilizational priority?

Toni Malet (Institut d'Història de la Ciència, Universitat Autònoma de Barcelona)

John Wallis on Symbolization and the Nature of Viète's "New Algebra".

Abstract: First published in the last decade of the 16th century, Viète's new understanding of algebra became increasingly popular in the 17th century, although the nature of Viète's new algebra and its historical pedigree remained contested. As a follow up of my previous talk on John Wallis (1616-1703), I shall focus now on specific features of Wallis's presentation of Viète's (and his early followers') new algebra, mostly drawing from his Treatise of Algebra (1685) but also taking advantage of other sources. I shall pay particular attention to Wallis' appropriation of Viète's unusual concept of species. An important novelty in mathematical thought c. 1600, the concept was to eventually usher in the 17th-century notion of mathematical symbolic variable. I will also discuss Wallis' presentation of the relationship between geometry and the new algebra. In particular I will look into Wallis's understanding of a substantial feature of the new algebra, the so-called logistica speciosa. It included a system of conventions ruling the formal, algorithmic manipulations of the signs representing mathematical magnitudes, arithmetical and geometrical. In this connection I shall discuss Wallis's understanding of the cogency of the algorithms ruling the formal manipulations of species.

Arilès Remaki (Johannes Gutenberg Universität Mainz)

Exponentiation: a notational issue?

Abstract: A parallel can be drawn between generalised exponentiation and the general multiplication of two quantities. In both cases, we are describing a process of generalisation that allows us to apply to any two terms a theory that was originally valid only on condition that one of the two terms of the operation was a natural integer, the operation then being interpreted as the iteration of another operation that makes a quantity behave with itself:

$$\underbrace{a + a + \cdots + a}_{n} = n \times a \qquad \underbrace{a \times a \times \ldots \times a}_{n} = a^n$$

While the first generalisation is a central point that has preoccupied a large number of historians of algebra, its 'multiplicative 'version occupies a rather minor place in historiography. In this more local context, Leibniz plays a role similar to that of Vieta. Not only is his work seen by historians as a decisive pivot towards generalisation, but this paradigm shift is also seen as a profound change in the use and conception of symbols in mathematics. More generally, Leibniz's mathematical work holds a special place in the historiography of symbols. Florian Cajori, who is interested in the history of mathematical characters, describes the German philosopher as the 'Master-builder of Mathematical Notations 'in an article of the same name. However, apart from this valuable work, which demonstrated from the early twentieth century the richness and diversity of Leibniz's symbolic practices,

the literature approached the theme of symbolism in Leibniz almost exclusively through the controversy surrounding the differential calculus. It was not until the end of the 20th century and Michel Serfati's thesis that Leibniz's generalisation of exponents was studied in detail. For Serfati, Leibniz's contribution was not simply part of the great symbolist movement initiated by Vieta, but in fact constituted a new founding figure of it. In his view, although symbolic algebra had suspended the nature of the quantities represented by the characters, it had not gone beyond the essentially quantitative nature of calculation. For Serfati, this overcoming, which he attributed to Leibniz in particular through the generalisation of exponents, constituted the true advent of symbolic thought. Unfortunately, Serfati's studies suffer from a very partial corpus. Many texts, unpublished at his time, undermine some of his analyses.

Leibniz himself produced numerous philosophical texts on the role of symbols and symbolic thought, in general and in mathematics in particular. Marcelo Dascal, in particular, explored this issue in the second half of the twentieth century. His work supports Leibniz's instrumental conception of symbols.

The aim of this presentation is to look back at these different works and to show how the authors' different conclusions derive from different conceptions of mathematical symbolism (instrumental, utilitarian or ontological).

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Ivahn Smadja (Nantes Université, CAPHI - Institut Universitaire de France (IUF))

Signs, symbols and operations: Humboldt on numeral systems and the historiography of mathematical symbolism

Abstract: “History of mathematics does not belong in my work”, Alexander von Humboldt cautioned in 1846, in the midst of a lively and fast-paced correspondence with Carl Gustav Jacob Jacobi, revolving around central issues in the warp and weft of the historiography of algebra. A self-proclaimed non-historian of mathematics, Humboldt earnestly engaged with the question of

mathematical symbolism twice – viz. at the beginning and at the end of his career, with a different focus and a different approach though. First, in the context of his path-breaking comparative investigations on numeral systems and place-value notation, from the late 1810s to the late 1820s; and second, in the late 1840s, while preparing the second volume of his monumental opus, *Kosmos*, as he struggled to navigate his way through the latest, often conflicting, views of Michel Chasles, Guglielmo Libri, or Georg Heinrich Ferdinand Nesselmann, on the nature, shaping and diffusion of algebra from its possible sources to the modern age.

In both cases, owing to a specific check-and-balance epistolary technique of his own devising, Humboldt created and orchestrated a goal-oriented dialogue between scholarly actors of various persuasion and domains of expertise, his own non-expert status qualifying him as a sounding board resonating with competing approaches. In both cases, he aptly circulated suggestions, queries and replies, engaging with different communities, whether with philologists, linguists and orientalists, or with mathematicians and historians of mathematics. Two such salient episodes of unequal consequence and significance will be highlighted and contrasted: his cross-examining exchanges with Silvestre de Sacy, Franz Bopp and Henry Thomas Colebrooke on the interpretation of dots, points and zeros in ancient numeral systems, on the one hand; and, on the other hand, his back-and-forth communication with Carl Gustav Jacob Jacobi and Peter Gustav Lejeune Dirichlet on the import of non-Western mathematical sources in the history of mathematics, and more specifically on when, where and how algebra started.

From one context to the next, however, Humboldt's viewpoint notably shifted from first-hand, methodologically innovative, comparativism to derivative historiography. His attitude also changed, veering from confidence to disbelief - yet, in both cases, though in different keys, questioning and keeping at bay the calibrating evidence of modern polynomial algebra. Humboldt's early endeavour, which he cherished and valued into old age, was to compare the largest range of numeral systems with a view to disclosing commonality of structure with place-value notation. His focus was on the spirit of the methods, rather than on the shape of the signs. And his comparative approach crucially depended on a tried-and-tested methodology based on a specific notational technology, the so-called "pasigraphic notations".

Only these would successfully achieve the sought non-reductive synoptic viewpoint on numeral systems, which was deliberately intended as different in kind from the abstractly unifying viewpoint readily afforded by polynomial algebra. Twenty-odd years later, while striving for a bigger picture, Humboldt incredulously put to the test some of the historiographical tenets of his time, asking his fellow mathematicians about the alleged divide between numerical and symbolic algebra, the relationship between algebra and analysis, or the nature of mathematical symbolism.

Although Humboldt did not explicitly link his early take on numeral systems with his late engagement with the historiography of algebra, our purpose with bringing together both ends is to contribute to delineating the so-called "retrospective historiography" of mathematical symbolism, through highlighting how certain actors bore witness to the process by which it took shape as mainstream historiography, while themselves thinking outside the box all along.

Ksenia Tatarchenko (JHU, Medicine, Science, and the Humanities program)
Reddening Signs: What is Marxist about the Russian Language Historiography of Mathematical Symbolism?

Abstract: This paper examines the Soviet corpus of textbooks and popular accounts in the history of mathematics, with a particular focus on the work of G. I. Bashmakova. In her 1979 popular science booklet, *The Making of Algebra*, Bashmakova offers an encompassing vision of algebra spanning Babylonian and modern-day applications. The booklet separates its narrative into distinct historical "turning" moments: the "birth" of symbolism in the work of Diophantus and its later European "creation." In the last chapter, devoted to François Viète, Bashmakova argues that while Viète's symbolism was the first to enable the replacement of certain mental operations with mechanical ones, this development was followed by a wave of advancements spanning mathematical analysis, vector and tensor geometry, the calculus of statements and sentences in mathematical logic, and "numerous other symbolic systems created to address specific mathematical and technical challenges." In other words, with the impressive print of 38,000 copies at the modest price of 11 kopeks each, the Soviet readers could appreciate the role of the history of symbolism in cultivating both the precision of human thought expression and the power of mental labor automation.

The central question of this paper is to investigate how the emphasis on symbolic computation as the language of modern mathematics—as in Bashmakova's work demonstrated above—relates to two defining phenomena underlying the rise of the Soviet school of the history of mathematics in general and the Soviet historical research of mathematical symbolism in particular: the first is Marxist philosophy, and the second is the pedagogical infrastructure that supported the professionalization of the community of historians of mathematics in the late Soviet era. I will use the example of Bashmakova's work on the history of algebra to ask how investigating pedagogical texts—specifically, how these texts present the history of symbolism and its relation to the Marxist framing of the nature of mathematical abstraction—allows us to challenge the received notion that the ideological context of planned science was an inherently limiting factor driving an "internalist" orientation in the Soviet history of science—one deemed too technical for censorship.

This paper is structured into three parts to test its working hypothesis using the case of the history of symbolism. First, in order to observe the change related to the pedagogical character of historical narratives about the development of symbolism in the 1970s, I examine the intellectual stakes of considering mathematical ideas in a long historical chronology, based on the early 1960s overview accounts focusing on the pre-modern history of mathematics and thus predating the initiative of the Soviet Ministry of Education to provide teaching materials in the history of mathematics and the widespread introduction of the history of mathematics into university curricula. These are exemplified by the two-volume project by Ernest Kolman and A. P. Yushkevich, which is systematically referenced by Bashmakova, including in the 1979 booklet, and therefore is constitutive of her historical vision of algebra's development across civilizational divides. Next, I will contextualize *The Making of Algebra* by comparing it to the 1976 handbook (also cited in the booklet's bibliography), produced by a team including Bashmakova and edited by Yushkevich. Finally, the paper will cross-check the censorship hypothesis by exploring the differences between Russian-language publications and the translations of works produced for

Western audiences. My aim is to determine when Marxist references are rhetorical and when they correspond to the Marxist framing of mathematical abstraction as well as a distinct methodology aiming at a canonical "broad, deep, and objective knowledge of the historical past." Such an understanding of history appears to generally fit Bashmakova's 1979 narrative about symbolism as a language of modern mathematics.

Benjamin Wardhaugh (University of Oxford)
Mathematical symbolism and its history in the Mathematical and Philosophical Dictionary *of Charles Hutton (1795/1815)*

Abstract: Charles Hutton (1737–1823) was a pivotal figure in the British mathematics of his generation, by virtue of his dense network of personal relationships, of his much-publicised antagonism to Joseph Banks and by extension the Royal Society, and of a series of magisterial publications including the *Mathematical Tables* (1785), the *Mathematical and Philosophical Dictionary* (1795) and the *Course of Mathematics* (1798). For several decades, his was the leading voice speaking about mathematics in English: and, through reprints and translations of his works, as well as through the long trajectories of his many friends, disciples and students, his views about the nature, function and historical development of mathematics remained widely influential into the middle decades of the nineteenth century.

This paper will examine Hutton's construction of a history of mathematical symbolism as articulated chiefly in his 1795 *Dictionary*. Hutton read widely in printed mathematics back to the early sixteenth century and followed closely the new publications of his British and European colleagues, as well as the papers in which Sanskrit mathematics was first presented to a European audience. But he also possessed a distinctive – and distinctively British – agenda, privileging a reliance on spatial and dynamic intuition, and on cognate forms of notation. I will attempt to show how Hutton used these materials to construct his particular – and influential – history of mathematical symbolism.

David Waszek
Uncontrolled symbolic computations in the history of the differential calculus and in non-Western mathematics: A first exploration of late 19th-century views

Abstract: In his *Elementarmathematik vom höheren Standpunkte aus* (1908; Engl. tr. 1932), Felix Klein included considerations on the history of mathematics. He theorized that three broad lines of development could be conceived as driving the growth of the discipline. Very roughly speaking, the first is logical, while the second is holistic and intuitive; as for the third, he wrote, it "has to do with what one denotes by the word *algorithm* (derived from a mutilated form of the name of an Arabic mathematician), that is, at bottom, *any kind of ordered, formal calculation*, in particular the *calculation with letters* [Buchstabenrechnung]." He further characterized the "algorithmic process" as "an autonomous, onward-driving force inherent in the formulas, operating apart from the intention and insight of the respective mathematicians, often indeed in opposition to them." To this third line of development, Klein then associated the beginnings of the infinitesimal calculus, for instance, but also "our modern system of numerals," ascribed to the Indians. This suggestive passage raises questions about the

relations, in the last decades of the 19th century, between two processes: on the one hand, the "rigorous founding of infinitesimal calculus," as Klein put it, accompanied by a reevaluation of the role of uncontrolled symbolic manipulations in its earlier history; on the other hand, a reconceptualization of the nature of non-European (especially Indian and Arabic) mathematical traditions. That the transformations of mathematics in the 19th century had a substantial impact on the way ancient sources were read has already been noticed in connection with the question of proof or rigor (e.g., Raina 2012; Smadja 2015). The goal of this talk is to pursue this line of inquiry with particular respect to symbolic computations and to connections between the historiography of European and non-European mathematics, in a late 19th-century German context.

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Zhou Xiaohan (Institute for the History of Natural Sciences, Chinese Academy of Sciences, Beijing)

Differences in the Understanding of Mathematical Symbolism Between Mathematicians and Historians of Mathematics in 19th and 20th century China

Abstract: In collaboration with British missionary Alexander Wylie (1815-1887), Li Shanlan (1811-1882), a prominent Chinese mathematician and translator of scientific works from English into Chinese, developed a novel mathematical symbolism for algebra and calculus, drawing on Chinese sources. The motivation and practice of such a creation highlight their views on the use of symbolism in English mathematical works and in ancient Chinese texts. Li Yan (1892-1963) and Qian Baocong (1892-1974), as two founders of the modern history of mathematics in China, offered commentaries on the symbolism in ancient texts, informed by their familiarity with modern mathematical symbolism. This talk develops a comparison of Li Shanlan's views with those of modern historians of mathematics, highlighting which aspects of symbolism these representative scholars emphasized or neglected. Qian Baocong once put forward the concept of "instrumental algebra", classifying certain methods in Chinese mathematical texts into this category. Li Shanlan and A. Wylie share a view that draws an analogy between the "position (*weici*)" in mathematical practices to which the Chinese documents attested and the "notation (*jihao*)" in algebraic texts they translated. Gaining insight into these differences and transformations in the way of viewing the history of mathematical symbolism is beneficial to forming our new comprehension of this issue.