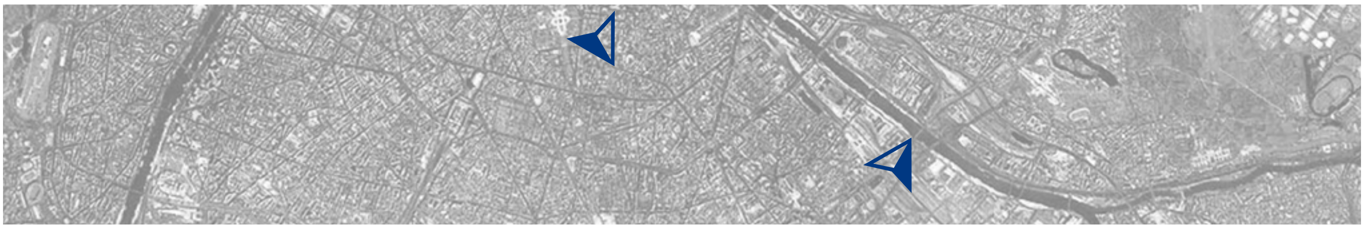

DIXIÈMES RENCONTRES FRANÇAISES DE PHILOSOPHIE DES MATHÉMATIQUES

FPMW 2018 10

PARIS 2018 / 11 / 8-10 10TH FRENCH PHILOSOPHY OF MATHEMATICS WORKSHOP

Il s'agit de la DIXIÈME ÉDITION du COLLOQUE ANNUEL DE PHILOSOPHIE DES MATHÉMATIQUES (FRENCH PHILMATH WORKSHOP) organisé, sous l'égide du GDR "Philosophie des mathématiques" (CNRS-INSHS), par une équipe de chercheurs français et étrangers.

10th INSTANCE OF ANNUAL PHILOSOPHY OF MATHEMATICS WORKSHOP (FPMW) organised, with the support of the GDR "Philosophie des mathématiques" (CNRS-INSHS), by a team of French and Foreign researchers.

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GdR PHILMATH

PROGRAMME

Jeudi 8 novembre / Thursday November 8

Université Paris Diderot,
salle 454A, bâtiment Condorcet, 4 rue Elsa Morante, 75013 Paris

09:00-9:30 Bienvenue et ouverture

09:30-11:00 René Guitart (IMJ PRG), *« Les scénarios : des esquisses aux topos et univers algébriques, l'analyse des théories par spécification des formes »*

11:15-12:30 Stefan Neuwirth (Univ. de Besançon), *" Philosophie de l'algèbre dynamique"*

12:30-14:00 Déjeuner

14:00-15:30 Rossana Tazzioli (Univ. de Lille), *"'Lezioni di Calcolo Differenziale Absoluto' de Levi-Civita"*

15:45-17:00 Benjamin Wilck (Humboldt Univ., Berlin), *"Les définitions mathématiques et un nouveau problème pour le scepticisme pyrrhonien"*

Vendredi 9 novembre / Friday November 9

Centre Cavaillès, ENS, salle 236, 29 rue d'Ulm, Paris 75005
(accès par le / access by 24 rue Lhomond)

09:30-11:00 Jean-Paul Van Bendegem (CLWF, Ghent Univ.), *"L'importance des trois points en mathématiques"*

11:15-12:30 Francesca Biagoli (Univ. of Vienna), *"L'approche transcendantale du raisonnement mathématique d'Ernst Cassirer"*

12:30-14:00 Déjeuner

14:00-15:30 Jean-Benoît Bost (Univ. Paris Sud), *"Les ambivalences de l'algèbre homologique"*

15:45-17:00 Sylvain Moraillon (Archives Henri Poincaré), *"La topologie de Poincaré à la croisée de la syntaxe et de la sémantique"*

Samedi 10 novembre / Saturday November 10

Université Paris-Diderot, salle 418C, bât. Halle aux Farines, esplanade Pierre Vidal-Naquet, 75013 Paris

09:30-10:45 Georg Schiemer (Univ. of Vienna), *"Deux façons de penser aux structures (implicites)"*

11:15-12:15 Deborah Kant (Univ. of Konstanz), *"Le forcing en tant que pratique ensembliste"*

Les scenarios : des esquisses aux topos et univers algébriques, l’analyse des théories par spécifications de formes.

René Guitart

Les théories sont classiquement spécifiées de façon logico-ensembliste, à l’aide de langages du premier ordre ou d’ordre supérieurs, ou via des monades. En fait on peut procéder autrement, court-circuitant l’approche logique, par spécification de limites, ou de recouvrements, ou de calculs relationnels algébriques, soit donc par les esquisses, les sites et topos, les univers algébriques.

Ce sont trois idées mêlées entre elles, et qui ont un point commun, celui de considérer des formes d’objets dans des catégories d'interprétations, formes au sens catégorique du terme (shape theory), et d’en spécifier des propriétés ou invariants.

On pourra appeler la spécification de propriétés des formes des « scénarios », et dire que les esquisses, topos et univers algébriques sont des scenari ou scénarios.

On a alors la notion de réalisation d’un scénario, et le problème fondamental est celui de la construction d'une réalisation librement engendré par une interprétation. En général ces modèles libres n’existe pas, dès lors que les scénarios ne sont pas algébriques, comme dans le cas des schémas récursifs de Herbrand.

Mais nous avons, dans certains cas, un théorème fondamental qui nous dit qu’il existe un Diagramme Localement Libre de réalisations (Guitart-Lair), et puis des théorèmes donnant des propriétés subséquentes des catégories de réalisations.

Le fait de voir les choses en termes de scénarios rend claire l’idée de remplacer la logique par la cohomologie, puisqu’il est possible de considérer la cohomologie en terme d’invariants de formes.

Philosophie de l'algèbre dynamique

L'algèbre dynamique est un programme de recherche dont le but est d'exhiber le contenu calculatoire des démonstrations de l'algèbre, et particulièrement lorsque celui-ci est rendu opaque par l'usage d'outils comme le lemme de Zorn.

Comme nous allons le voir sur un exemple paradigmatique, ce travail est de nature à fournir une perspective nouvelle sur les concepts en jeu à la fois dans l'énoncé d'un théorème que dans le déroulement de sa démonstration. En effet, il amène à forger de nouveaux objets qui reflètent les calculs réalisés.

Ce programme de recherche a plusieurs versants. Un premier est de nature purement algébrique et correspond à un travail de clarification. Un deuxième, de nature logique, est une réflexion sur la forme des raisonnements et donne lieu à l'étude des « théories géométriques du premier ordre », ou « théories cohérentes », et des « théories géometriques infinitaires » ; cette réflexion consiste paradoxalement à débarrasser les calculs de la logique. Un troisième s'inscrit dans la théorie des catégories et fait le lien avec les topos de Grothendieck.

Un exemple paradigmatique

Les entiers relatifs, $\mathbb{Z}$, donnent lieu à un corps de fractions, $\mathbb{Q}$, dont le groupe multiplicatif, $\mathbb{Q} \setminus \{0\}$, forme, au signe près, le *groupe de divisibilité de l'anneau d'intégrité* $\mathbb{Z}$, dont les éléments sont ordonnés par la relation de divisibilité $\mid$: on dit que $q \mid r$ si $r/q \in \mathbb{Z}$. Cet ordre n'est pas total : il se peut qu'on ait à la fois $q \nmid r$ et $r \nmid q$, par exemple $1 \nmid \frac{3}{2}$ et $\frac{3}{2} \nmid 1$. Cependant, le groupe de divisibilité peut être plongé dans un produit cartésien de groupes totalement ordonnés, grâce à la décomposition unique d'un nombre rationnel non nul q comme produit de facteurs premiers : q s'écrit $q = \pm 2^{\alpha_2} 3^{\alpha_3} 5^{\alpha_5} 7^{\alpha_7} 11^{\alpha_{11}} \cdots$, et on peut envoyer q sur $\varphi(q) = (2^{\alpha_2}, 3^{\alpha_3}, 5^{\alpha_5}, 7^{\alpha_7}, 11^{\alpha_{11}}, \cdots)$ dans le produit cartésien des puissances de 2, de 3, de 5, de 7, de 11, etc., chacun totalement ordonné par divisibilité. En d'autres mots, l'ordre de divisibilité est une conjonction d'ordres totaux.

La conséquence pratique de ceci est que tout calcul dans le groupe de divisibilité correspond à ce même calcul dans chacun de ces groupes totalement ordonnés de puissances de nombre premier, et que si ce calcul aboutit au même résultat dans chacun de ces groupes totalement ordonnés, ce sera encore le cas dans le groupe de divisibilité.

Le théorème fondamental des anneaux intègres de Krull donne la contrepartie abstraite de cette observation. Mais dans l'énoncé de ce théorème, la contrepartie des nombres premiers, les « valuations », résultent d'une application du lemme de Zorn. Intuitivement, cela se voit dans l'argument ci-dessus par le fait que les nombres premiers tombent du ciel.

C'est Paul Lorenzen (1915-1994) qui a clarifié ce théorème et en a révélé la nature dynamique. Celle-ci est que pour tout calcul dans le groupe de divisibilité d'un anneau d'intégrité « intégralement clos », on peut toujours *faire l'hypothèse que les éléments du groupe qui apparaissent dans le calcul sont totalement ordonnés.* Faire cette hypothèse rend le calcul *dynamique* au sens suivant : pour deux éléments q et r du calcul, on a le droit d'ouvrir deux branches dans ce calcul. Dans la première, on suppose que $q \mid r$; dans la deuxième, que $r \mid q$. Si on aboutit au même résultat dans les deux branches, alors on aboutit à ce résultat aussi sans avoir fait cette hypothèse.

Analyse philosophique

Nous commençons par fournir une description du cheminement et des motivations de Lorenzen, sur la base de la correspondance inédite avec Wolfgang Krull (1899-1971).

Le cadre ontologique dans lequel se place le théorème fondamental des anneaux intègres de Krull est celui de la théorie des ensembles dans une forme actualiste : le lemme de Zorn aboutit à l'existence des valuations parce que l'univers des objets étudiés est supposé exister au préalable. En d'autres mots, des définitions imprédicatives donnent lieu à l'existence des objets recherchés.

Quel est le cadre ontologique de l'algèbre dynamique ? C'est celui des « définitions inductives ». Plutôt que de garantir un cadre immuable pour toute preuve à venir, les définitions inductives construisent le cadre d'un calcul de manière concomitante avec ce calcul. La possibilité de ces définitions résulte à chaque fois du constat d'une structure arborescente qui n'admet pas de détour, ou dont tout détour est éliminable. Il s'agit donc d'une ontologie dynamique, dans laquelle les objets ont la même plasticité que les raisonnements.

Nous conclurons sur la manière dont les deux cadres ontologiques s'éclairent réciproquement.

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Les traités de Tullio Levi-Civita sur le calcul tensoriel. Enjeux scientifiques, difficultés pédagogiques et circulation du savoir mathématique

Rossana Tazzioli
Laboratoire Paul Painlevé, Université de Lille

Le traité *Lezioni di Calcolo Differenziale Assoluto* de Tullio Levi-Civita, publié à Rome en 1925, a été traduit en anglais en 1927. Cette traduction est en fait beaucoup plus riche que la publication italienne car elle contient d'autres sujets, comme les applications à la physique, et aussi un nouveau concept mathématique, essentiel en géométrie différentielle et en théorie de la relativité : la déviation (ou l'écart) géodésique. Dans mon exposé, je voudrais aborder les questions suivantes:
Quelle méthodologie a suivi Levi-Civita pour rassembler plusieurs concepts de « calcul différentiel absolu » (le calcul tensoriel moderne) et les rendre cohérents dans une théorie adaptée aux étudiants? Ce traité peut-il aider l'historien des sciences à mieux comprendre l'émergence du calcul tensoriel, et comment ce dernier a contribué à la mise en place de concepts nouveaux en géométrie différentielle et en physique mathématique?
Enfin, quel fut le rôle des traités de Levi-Civita pour la circulation du calcul tensoriel?
Dans cet exposé j'utilise les lettres de l'Archive Levi-Civita, conservées à la Bibliothèque de l'Accademia dei Lincei à Rome.

Mathematical Definitions and a New Problem for Pyrrhonian Scepticism

Résumé

My paper identifies a previously unnoticed problem for the application of Pyrrhonian scepticism to scientific principles, in particular geometrical definitions. In the *Outlines of Scepticism* (book I, sections 8–10), Sextus Empiricus defines his sceptical method as an ability to suspend belief about any given proposition by constructing pairs of opposing and equally convincing arguments. In *adversus Mathematicos* (= *M*) I–VI, Sextus nonetheless presents a series of straightforward refutations of scientific doctrines rather than oppositions of arguments and counterarguments, even though Sextus programmatically announces at the beginning of *M* that he is going to argue by way of sceptical oppositions. That's why commentators have thought that the method deployed in *M* I–VI is not Pyrrhonian scepticism, but is rather negative dogmatism (Pappenheim, 1874: 16–17; Apelt, 1891: 258–259; Zeller, 1923: 51n2; Janáček, 1972; Russo, 1972: viii n2; Pellegrin *et. al.*, 2002: 23–24; cf. Barnes, 1988: 76–77; Desbordes, 1990: 169). Recently, however, it has become widely accepted among scholars that the apparent lapse from Pyrrhonian scepticism into negative dogmatism, which we find in *M* I–VI, can be rectified by supplementing additional arguments opposing Sextus' refutational arguments (Blank, 1998, I–IV; Desbordes, 1998: 168; cf. Barnes, 1988: 72–77; Morison, 2004: section 5).
Against this I present a counterexample. While the aforementioned strategy accounts for scientific theorems, which are usually accompanied by a proof, it fails in the case of definitions, for which there is no proof or justification of some other sort. The arguments in *M* I–VI against particular scientific definitions cannot, therefore, be instances of Pyrrhonian scepticism. Neither the standard (Annas and Barnes, 1985: 24; 39; 82–83; 98; 102; 121–122; cf. Striker, 1983: 100; Hankinson, 1995: 159) nor the most recent (Morison, 2011) interpretations of Pyrrhonian scepticism give a satisfying interpretation of Sextus' arguments against particular definitions. Hence, although Pyrrhonian scepticism is supposed to be applicable to all kinds of proposition or belief, there turns out to exist one type of proposition or belief to which it does not apply, namely scientific definitions.

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The importance of three dots in mathematics

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We are all familiar with the use of the three dots (…) in mathematics (as we are in ordinary language). In the very same way that Menger (1956) showed the delicate and complicated use of variables in mathematics, it can be shown that three dots have multiple uses and meanings. Against the argument that, in order to avoid such difficulties, the three dots can always be eliminated, I will show how a formalization can be given (on the propositional level) of the (use of) three dots. In terms of mathematics education, this analysis of '…' shows that it is anything but a trivial task for pupils to grasp its diverse meanings and uses. In terms of the philosophy of mathematics, this presentation can be seen as an exercise in the study of mathematical practices (see Van Bendegem (2018) for an overview, but see also Ferreiros (2015)).

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Ernst Cassirer's Transcendental Account of Mathematical Reasoning

Cassirer's philosophical agenda revolved around what appears to be a paradoxical goal, that is, to reconcile the Kantian explanation of the possibility of knowledge with the conceptual changes of nineteenth and early twentieth-century science. This paper offers a new discussion of one way in which this paradox manifests itself in Cassirer's philosophy of mathematics. Beginning in 1910, Cassirer articulated a unitary perspective on mathematics as an investigation of structures independently of the nature of individual objects making up those structures. However, a tension remains between Cassirer's demand for the unity of knowledge and his reliance on the structural methods of nineteenth-century mathematics. Cassirer tried to resolve this tension in his early works by pointing out that the loss of unity with regard to the subject-matter of modern mathematics – insofar as this ceases to define itself as the science of numbers and quantities – is compensated by the deeper unity of its method. However, after the development of modern axiomatics, Cassirer realized ever more clearly that mathematics (including the most abstract parts of it) raises new problems of its own. In general, beginning in the 1920s, he acknowledged different types of objectivity at stake in the different ways to understand the world, which he called "symbolic forms." Limiting the consideration to epistemology, it seems that in order to account for the unity of mathematics in the latter sense, it would be inevitable to call into question the unity of knowledge in Cassirer's original account.

More recent discussions of Cassirer's philosophy of mathematics reflect the same tension. Jeremy Heis suggests that a charitable way to read Cassirer today would have to offer a unitary account of mathematical objectivity. By contrast, Thomas Mormann maintains that the central thesis of Cassirer's philosophy from 1910 to his later works is that mathematical and physical knowledge are of the same kind (*sameness thesis*). In order to spell out what the sameness thesis entails, Mormann offers a series of examples of how the extension of both kinds of knowledge requires the introduction of ideal elements. It follows that a consistent development of the sameness thesis in the light of twentieth-century mathematics would have to acknowledge incompatible idealizations. In other words, quite contrary to Heis, Mormann's suggestion is to allow for a plurality of conceptual frameworks in the philosophy of mathematics in order to retain the main insight of the sameness thesis.

This paper aims to clarify how both aspects of Cassirer's philosophy stand together by drawing attention to the transcendental argument at stake with the sameness thesis. By transcendental here I mean all kinds of arguments that set conditions for the possibility of knowledge. In particular, the argument under consideration reflects the structure of a transcendental deduction in Kant's sense: in order to justify the possibility of knowledge, Kant offers a proof that the fundamental concepts of the understanding necessarily apply to the manifold of intuition. According to Cassirer, the logic at work in the formation of numerical concepts necessarily applies to spatial concepts and spatiotemporal relations. This offers an explanation of the extensibility of mathematical knowledge from abstract to empirical domains.

I will contend that Cassirer's argument derives from the reading of the Kantian theory of space articulated by Cassirer's teachers, Hermann Cohen and Paul Natorp, and further developed by Cassirer in the second volume of *The Problem of Knowledge in Modern Philosophy and Science* (1907). I will then turn to how Cassirer connects the account of mathematical reasoning that emerges from this reading to the structuralist methodology of nineteenth-century. My suggestion is that a more careful consideration of the key examples for Cassirer's account can shed light on his long-term strategy to resolve the tension between his emphasis on the unity of mathematics and the sameness thesis. Mathematical and structural reasoning typically include the embedding of a particular domain into a larger structure. Paradigmatic examples of this are the introduction of irrational numbers as limits of converging series of rationals and the generalization of the Euclidean plane to the projective plane. While these examples underpin a unitary perspective on specific mathematical disciplines, I will contend that Cassirer emphasized a no less essential aspect of mathematical concept formation, that is the transposition of structural methods from one specific domain to another. Three examples are particularly relevant here: (1) Richard Dedekind's definition of natural numbers, (2) Felix Klein's use of transfer principles, (3) the construction of a numerical scale on the projective line. These are examples of how structural procedures are transferred across algebraic, numerical and geometrical domains. At the same time, they lend plausibility to Cassirer's argument about the extensibility of such procedures to empirical domains in a unitary but internally articulated view of knowledge.

My suggestion is that Cassirer offers a philosophical account of cases where structural reasoning finds unexpected applications beyond the original ground for its development.

Les ambivalences de l'algèbre homologique

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L'algèbre homologique, entendue dans son sens le plus large, est depuis ses origines l'objet de jugements contradictoires concernant sa difficulté technique, le caractère significatif de ses méthodes, sa place dans l'architecture des mathématiques et dans la formation des chercheurs, etc. Dans cet exposé, je présenterai plusieurs de ces ambivalences, dans une perspective historique, et je m'interrogerai sur leur signification.

Poincaré’s topology at the interplay between syntax and semantics

Mark Wilson proposes an original account of the grip our linguistic tools give us on the world, as illustrated in science and in mathematics. By studying for instance the development of projective geometry, Wilson describes a conflict between what he calls an apparent and an active grammar, reflecting the seemingly contradictory requirements of a powerful syntax and a coherent semantics. Wilson’s originality lies in his refusal to see this duality as a transitory aspect of science, to be dismissed by the adoption of a proper and definitive setting ; rather, he emphasizes the necessity of long periods of « semantic agnosticism » in the emergence of certain mathematical fields, in order to be able to reap the inferential benefits of new syntactic tools, long before engaging in a systematic semantic clarification of this particular domain ; what is more, this clarification is never assured to be definitive ; in fact, we have to consider this opposition as a productive dialectic that is pervasise in human conceptual behavior.

We would like to study some parts of Poincaré’s work on topology (a field he contributed to create to an essential degree) according to Wilson’s lens. But instead of focusing on the syntactic requirement by which Wilson characterizes the emergence of a new discipline, we think that Poincaré is preoccupied in turn with both the syntactical and the semantical requirements, so that his « Analysis situs » exhibits in a compressed manner the dialectic Wilson describes in the longer run. In the introduction of his first memoir, Poincaré expresses a semantical concern ; he conforms itself to a tradition going back to Poncelet by making the case for a geometrical language, better equipped to give us comprehension as an analytical one. Nevertheless, as far as language is concerned, Poincaré’s main innovation is algebraic ; a fact exemplified for instance with the introduction of the homologies, which gives us a powerful syntactic tool, by simplifying steps necessary to define topological invariants. But as the same time it exerts a shift in the geometrical interpretation of topological objects.

The most dramatic evidence of a tension between an apparent and an active grammar is then to be found in the conflict between Poincaré and Heegard about the duality theorem. At the root of the opposition between the two mathematicians lies a disagreement about the range of the operations permitted by the new formalism. In order to answer Heegard’s critique and save the validity of this essential piece of topology, Poincaré is compelled, in a reply to Heegard and then in the first complement to the «Analysis situs », to extend the power of his algebraic tool, once more at the cost of the geometrical interpretation ; but he was not satisfied with this loss in semantics and strived to provide a new one ; which he did soon afterwards in the second complement , giving rise to the new and fruitful notion of torsion.

According to Poincaré, topology was a central piece of mathematical knowledge, a fact he explained by its exhibiting geometrical intuition in its purity ; we think that it can be explained, thanks to Wilson’s conceptuality, in a more complex manner, which can in turn explain Weyl’s opinion according to which « the angel of topology and the devil of abstract algebra fight for the soul of every individual discipline of mathematics. »

Two ways to think about (implicit) structure

According to a dominant view in modern philosophy of mathematics, mathematics can be understood as the study of abstract structures (e.g. [2], [8]). Put differently, structuralism holds that theories of pure mathematics (such as Peano arithmetic, lattice theory, topology, or graph theory) study only the structure or the structural properties of their respective subject fields (namely number systems, lattices, topological spaces, and graphs). But what precisely is the relevant structure of such mathematical entities? How can we think about their structural content? The present talk will address these questions based on a distinction between *primitive* and *implicit* structure, that is, between systems defined by mathematical theories and their implicit structural content.

As is well-known, mathematical objects such as lattices or topological spaces are usually introduced axiomatically today, that is, in terms of formal axiomatic definitions that specify the constitutive properties of the objects in question. For instance, a topological space can be defined in terms so-called neighborhood axioms, first introduced by Felix Hausdorff in 1914, which specify the properties of a neighborhood relation between points and sets of point sets. While it is clear that the primitive properties of mathematical systems are specified axiomatically in this sense, less has been said about how mathematicians investigate the *implicit* structure of such systems. In this talk, we will compare two general ways to think about this implicit structural content of theories of pure mathematics. According to the first approach, the implicit structure or the structural properties of mathematical objects are specified with reference to formal languages, usually based on some notion of definability. Thus, properties of systems such as rings or graphs will count as structural if they are logically definable in a formal language of the correct mathematical signature. According to the second approach, structures are determined in terms of invariance criteria. For instance, the structural properties of a given mathematical system are often said to be those properties invariant under certain transformations of the system or under mappings between similar systems (see [1], [6]). In the talk, we will investigate these two approaches by drawing to a particular mathematical case study, namely the study of simple incidence structures in *finite* affine and projective geometry.

Given these geometrical examples, we give a philosophical analysis of the conceptual differences between the two methods to express implicit structure. The talk will focus on three issues. The first concerns the conceptual motivation for treating mathematical structures in terms of the notions of definability and invariance. Why are these criteria adequate means for the specification of structural properties? As will be argued, both methods capture some form of "topic neutrality" underlying the structuralist account of mathematics. In the case of invariance, this is due to the fact that mathematics is indifferent to the intrinsic nature of mathematical objects and thus also indifferent to arbitrary switchings of such objects in a given system. In the case of definability-based approaches, the relevant topic neutrality is related to the fact that adequate logical definitions should be reducible to statements about the primitive mathematical structure. The structural topic neutrality in mathematics thus seems to be explainable in terms of the “formality” of logic (see [3]).

The second point addressed in the talk concerns the logical relation between two approaches to implicit structure. As work in logic and model theory has shown, the exist a general *symmetry* or *duality* between the method of specifying invariants relative to transformations or mappings and the notion of definability (see, in particular, [5]). We will present a formal account of this duality in terms of a Galois connection between automorphism classes and Galois-closed sets of relations. More specifically, given this framework, it will be shown that the class of definable properties (of the objects) of a given primitive structure always forms a subclass of the class of properties invariant under the automorphisms of the structure.

Finally, we discuss the relevance of the two ways to think about implicit structure for our understanding of mathematical structuralism. Here, in particular, the focus will be on the notion of the *equivalence* of mathematical structures (see [7], [8]). Building on the existing literature on the topic, we will propose two notions of structural equivalence that take into account not only the (axiomatically defined) primitive structure, but also its implicit structural content. The first notion is motivated by the idea of definable implicit structure and based on the notion of *interpretability*. According to it, two mathematical structures are equivalent if they are bi-interpretable (compare [4]). The second notion, in turn, is motivated by the invariant approach and based on the concept of ”transfer principles” between structures. According to it, two structures are equivalent if there exists a mapping between their domains that induces an isomorphism of the respective automorphism groups.

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Forcing as a Part of Set-Theoretic Practice

July 1, 2018

In mathematics, statements can be proven to be true, or they can be refuted and then are proven to be false. In set theory however, statements can be proven to be neither provable nor refutable. This fact is called the set-theoretic independence phenomenon, which gives rise to the philosophical independence problem: *What is the status of a statement which is neither provable nor refutable?*

The most powerful method for such independence proofs is the method of forcing. Since more than fifty years, forcing is a part of set-theoretic practice. Much set-theoretic knowledge of today is based on forcing, and the independence problem—the main problem in the philosophy of set theory—is as severe as the forcing method is successful in application.

In order to approach the philosophical independence problem, the role of forcing in set-theoretic practice is investigated. As a suitable method for this descriptive investigation, the analysis of several interviews with expert set theorists is chosen.

Different aspects of the role of forcing are examined:

Spread of forcing in the set-theoretic community It is asked to what extent today's set-theoretic practice is determined by forcing. There are contrasting hypotheses on that question. One could have the view that set theory today is actually the theory of the models of set theory, in which the forcing technique is a key method. However, this view is challenged by the fact that there are set theorists who never use forcing. An appropriate description of the spread of forcing in set theory is given.

Naturalness of forcing at its introduction and today The following hypothesis is considered: Forcing was unnatural at its introduction, and today it is a natural part of set-theoretic practice. The introduction of forcing in set theory by P. Cohen is mostly considered a surprising event.

But one could also have the view that in the 60's, the time was ripe for the forcing result on AC and CH. The question of the naturalness of forcing today is connected to the first topic. Set theorists who use forcing regularly might find forcing very natural, while set theorists who never use forcing might not agree on that judgement. A refined evaluation of the naturalness of forcing is given.

Compatibility of the use of forcing and the universe view The use of forcing might better support a multiverse view than a universe view. For Hamkins, the widespread use of forcing is one essential argument for a multiverse view. But there are also set theorists with a universe view who use forcing. Different possibilities how the use of forcing can be compatible with the universe view are presented.

Acceptability and use of forcing axioms The question how forcing axioms are used, and which of their properties are important in connection to the question of acceptability is the main question for this topic. Large cardinal axioms seem to be better candidates for acceptability than forcing axioms. But there are set theorists who say that forcing axioms are natural because they correspond to the idea of a forcing-saturated universe. One could also say that the property of consistency strength weighs much more with regard to acceptability than the difference between large cardinal axioms and forcing axioms. Different views on forcing axioms, and kinds of using them are presented, compared to the case of large cardinal axioms and evaluated with regard to the question of acceptability.

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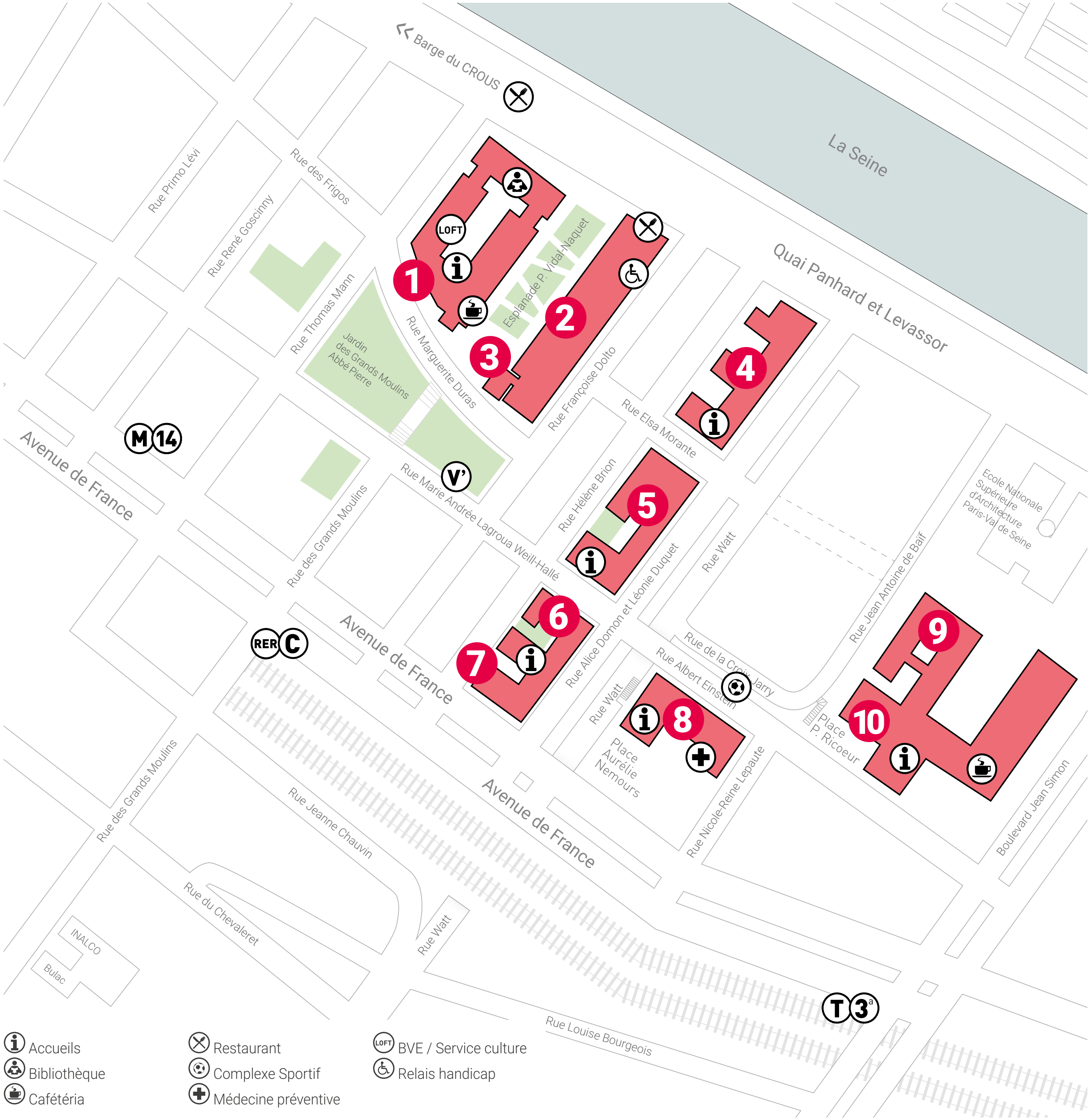

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