# A New Symbolic Writing for Encoding String Figure Procedures

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**Abstract**

In this article, we present a new symbolic writing for encoding string figure processes. This coding system aims to rewrite the string figure procedures as mathematical formulae, focusing on the operations implemented through these procedures. This formal approach to string figure making practices has been developed in order to undertake a systematic comparison of the various string figure corpora (at our disposal), and their statistical treatment in particular.

**Key Words**: String figures, operations, procedures, coding system, modeling.

## 1 Introduction

From the first scientific studies on string figure-making practices—from the 19th century and onwards—a number of anthropologists have attempted to devise suitable and efficient nomenclatures to record the procedures involved in the making of such figures. The first published nomenclature by Cambridge anthropologists William H. R. Rivers and Alfred C. Haddon (1902), has been used and refined by ethnographers like Diamond Jenness (1886-1969), Honor Maude (1905-2001), James Hornell (1865-1949), and many others who have collected string figures in different societies throughout the 20th century. This method of writing down string figures processes has been further standardized thanks to the work of string figure experts (like

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Mark Sherman, Will Wirt, Philip Noble, Joseph D'Antoni, among others), all of them members of the International String Figure Association (ISFA). This writing system—known as the ISFA nomenclature—is currently the most commonly used method to write down string figure procedures.

In parallel, a few mathematicians have been interested in devising symbolic notations in order to encode—in a more synthetic way, as a mathematical formula—the sequence of operations that are implemented in string figure making processes. In particular, in the 1980s, mathematician Thomas Frederick Storer (1938-2006) published a long article in which he developed several mathematical approaches for encoding and analyzing string figure processes (Storer, 1988) [1]. One of these approaches—the "Calculus for string figures"—has been devised in order "to conserve string figures uniquely, in a manner not heretofore possible, through the development of an unambiguous formal language for their discussion" (Storer, 1988, p. i).

Storer's work has been of fundamental importance for my own investigation on the topic. Indeed, it is by combining Storer's Calculus and the symbolic notation devised in my previous studies on string figures (Vandendriessche, 2010)—that the new coding system presented in the present paper has been imagined and created.

In the latter studies, I have analyzed a string figure process as a series of "elementary operations" (or "simple movements")—in the sense that the making of any string figure of a given corpus can be described by referring to a certain number of these operations. A string figure can thus be regarded as the result of a procedure consisting of a succession of elementary operations (Vandendriessche, 2008). With the goal of carrying out a comparative and statistical analysis of the occurrence of the elementary operations—involved in different string figure corpora, we have been led by the idea of creating a symbolic system for encoding string figure procedures permitting to easily isolate (within the whole process) each of these elementary operations.

Furthermore, rewriting string figure corpora using a mathematical/symbolic notation should enable us to better identify and analyze the different ways in which the string figure practitioners have combined these "elementary operations" in what I have suggested calling "sub-procedures"—*i.e* any succession of elementary operations either shared, or used in the same way in several string figure procedures, or iterated in the same one. In that perspective, a digital tool should be elaborated in order to identify and automatically bring to light the various (encoded) sub-procedures of a given corpus. This would allow a comparative analysis—on that aspect—from a string figure corpus to another.

[1] See also (Vandendriessche, 2015b).

Working in this way, we might expect—as Storer previously wished—"to explicate the structure underlying the set of all string-figures by exploring their interrelations" (Storer, 1988, p. i).

# 2 Description of the Symbolic Writing

## 2.1 Labeling the Functors

As in Thomas Storer's "Systemology" (Storer, 1988), Fingers are numbered from 1 (thumb) to 5 (little finger). $R$ and $L$ indicate "right hand" and "left hand" respectively. In this way, $R1$ is the notation of the right thumb, whereas $L2$ denotes the left index.
The ten fingers are sometimes used with the teeth ($T$), a big toe ($G$) or the wrist ($W$). All of these have been termed "Functors" by Storer.

| **Summary table - Functors** | |
|---|---|
| **Symbols** | **Definition** |
| 1 | Thumb |
| 2 | Index |
| 3 | Middle finger |
| 4 | Ring finger |
| 5 | Little finger |
| $Ri$ | $i^{th}$ finger of the right hand |
| $Li$ | $i^{th}$ finger of the left hand |
| $R, L, B$ | Right Hand, Left Hand, Both Hands |
| $T$ | Teeth |
| $G$ | Great toe |
| $W$ | Wrist |

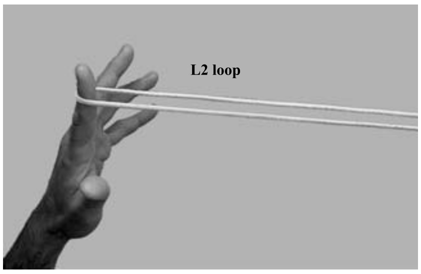
## 2.2 Labeling the Objects

The Functors operate on what Storer calls "Objects". The Objects are separated into two groups: "strings" and "loops". A loop is formed when the string passes around a finger. Picture 1a shows a loop made on the left index $L2$.

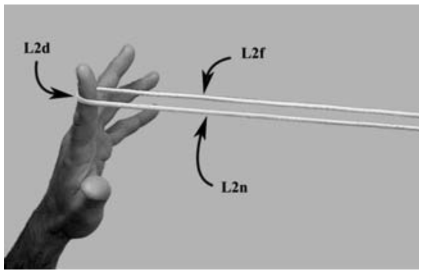
### 2.2.1 Strings or loops carried on fingers

A loop is denoted by using the symbol "$\infty$". When $i \in \{1,2,3,4,5\}$, $Li\,\infty$ symbolizes the loop made on the $i^{th}$ finger of the left hand (for example, the loop on pictures 1a-b will be noted $L2\infty$). In the same way, $Ri\,\infty$ will define a loop made on the $i^{th}$ finger of the right hand.

The "string" making the loop is divided into three parts. The one which lies on the "dorsal" side of the finger—cf. (Rivers & Haddon, 1902)—is written symbolically *Rid* or *Lid* for a loop made on the $i^{th}$ finger. The "near" string of a loop (made on a finger) is the string closest to the practitioner, when the finger is pointing up and the palm is perpendicular to the practitioner's chest. The third one is denoted as the "far" string[2](picture 1b).




1a- Loop formed on the left index




1b- $L2\infty$ and its strings

The notations will be the following:

- $Rin$: right near string on the $i^{th}$ finger
- $Rif$: right far string on the $i^{th}$ finger
- $Lin$: left near string on the $i^{th}$ finger
- $Lif$: left far string on the $i^{th}$ finger

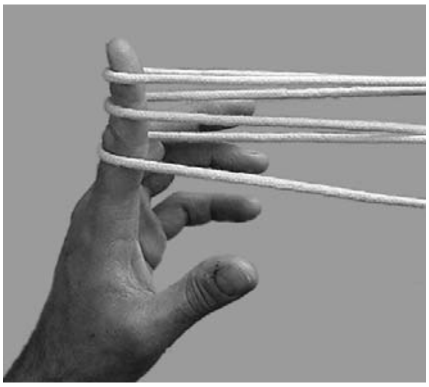
It frequently happens that several loops are carried by the same finger. In such a case, Storer uses the following notations: "If, in a given string position the generic finger, $F$, has the generic natural number $n$ loops we name these—beginning at the base of $F$ and proceeding to the tip—as follows:" (Storer, 1988, p. 21).

The symbols *l, u, m* are used as the abbreviation of *lower*, *upper* and *median* respectively. However, I did not find, either in the anthropological literature or in my own fieldwork findings, a configuration in which a finger carries more than 3 loops (picture 1c.).

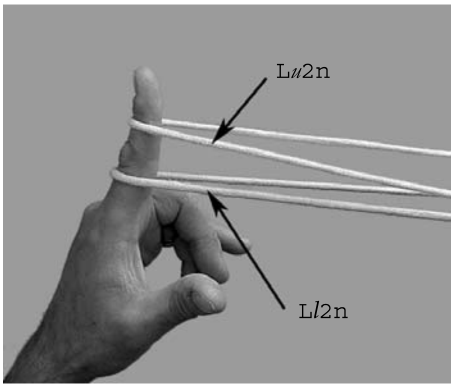
The same symbols *l, u* and *m* will be used to differentiate the different strings that run from one given finger. For instance, when the index finger $L2$ carries two loops, two different near strings start from this finger. They will be noted $Ll2n$ and $Lu2n$ (picture 1d).

[2]The meaning of "near" and "far" in this context is equivalent to Rivers & Haddon's definition of "radial" and "ulnar" respectively (Rivers & Haddon, 1902, p. 147)

| $\underline{n}$ | $\underline{F\infty' s}$ |
|---|---|
| 1 | $F\infty$ |
| 2 | $lF\infty, uF\infty$ |
| 3 | $lF\infty, mF\infty, uF\infty$ |
| 4 | $lF\infty, m_1F\infty, m_2F\infty, uF\infty$ |
| 5 | $lF\infty, m_1F\infty, m_2F\infty, m_3F\infty, uF\infty$ |
| . | |
| . | |
| . | |
| $n$ | $lF\infty, m_1F\infty, m_2F\infty, ..., m_{n-2}F\infty, uF\infty$ |



1c- Three loops on $L2$




1d- Lower and upper left index near strings

- $x\infty^n$ denotes the $n$ loops carried by the fingers $x$. In the example above, the two loops carried by the indices can be encoded $2\infty^2$.

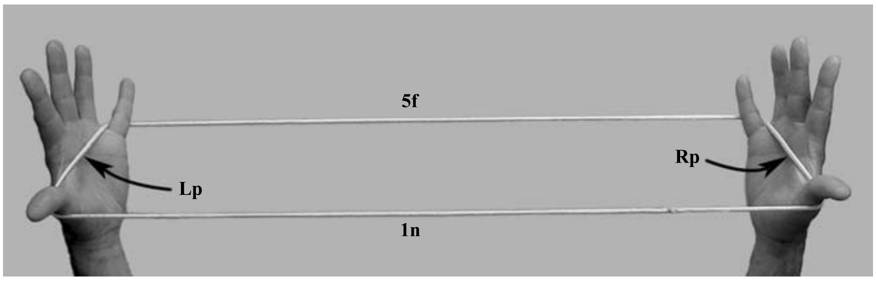
**Remark**: It sometimes happens that a particular loop—after having been released—is temporally carried by another part of the body or the string configuration itself. In this case, we will use the code *ex.*, followed by the last coding of the loop in question. For instance, $ex.Ri\infty$ will mean: the loop which was previously carried by the $Ri$ finger. See the example in section 3.2.

### 2.2.2 Transverse strings

It happens sometimes that a string cannot be easily defined as carried by a finger. When such a string crosses the configuration from one hand to the other, perpendicular to both palms (when they face each other), it is called a "transverse" string and encoded *tv*. When several such a transverse strings are created, they can be differentiated by using the terminology *l, u, m* and *n, f* defined above.

## 2.3 Labeling the Starting Positions

Anthropologists Alfred Cort Haddon and William H. R. Rivers defined "Position I"–that we will encode *P.I*–as the initial position obtained when loops are formed on the thumb and little finger of both hands[3]. In this case, the left and right palmar strings (a string which lies on the palm of the hand) are then created (picture 2). These two palmar strings will be denoted as *Lp* and *Rp*.




2- Position I

As shown above, the string connecting $L5f$ to $R5f$ and the one between $R1n$ to $L1n$ will be noted simply $5f$ and $1n$ respectively.

[3]Cambridge anthropologist Alfred Cort Haddon (1855-1940), in collaboration with anthropologist, neurologist and psychiatrist, William H. R. Rivers (1864-1922), carried out the first significant study on string figures. In 1902, they published an article entitled "A Method of Recording String Figures and Tricks" (Rivers & Haddon, 1902). In this paper the authors explained their methodology for collecting string figures and tricks. Moreover they proved the efficiency of their nomenclature by writing down the making of twelve Melanesian string figures that had been collected during the 1898-99 Cambridge expedition conducted by Haddon in the Torres Straits. The goal of the article was clearly to induce their colleagues to pay attention to the topic and help them to collect string figures in their own fields. For Haddon & Rivers' definition of "Position I", see (Rivers & Haddon, 1902, p. 148).

### 2.3.1 Summary

| Summary table - Objects | |
|---|---|
| **Symbols** | **Definition** |
| **Loops** | |
| $Li\infty$ | Loop held by $i^{th}$ finger of the left hand |
| $Ri\infty$ | Loop held by $i^{th}$ finger of the right hand |
| $W\infty$ | Loop on the wrist |
| **Strings** | |
| $Lif$ | Far string of the loop held by the finger $Li$ |
| $Rin$ | Near string of the loop held by the finger $Ri$ |
| $Lid$ | Dorsal string of the loop held by the finger $Li$ |
| $if$ | Entire string encompassing the connected $Lif$ and $Rif$ |
| $in$ | Entire string encompassing the connected $Lin$ and $Rin$ |
| $Rp$ | Right palmar string |
| $Lp$ | Left palmar string |
| $tv$ | Transverse string |

Objects and Functors are now well defined. In order to complete this symbolic writing for string figures, we now have to devise a symbolization of the elementary operations, analyzed as a given Functor (fingers, teeth, feet) operating on one or several Objects (strings, loops).

## 2.4 Encoding Elementary Operations

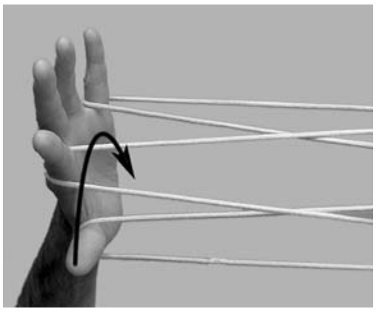
### 2.4.1 Operation "Inserting"

The operation "inserting" (a finger into a loop) is encoded by letter "$i$" and either an "upper bar" or a "lower bar" in order to indicate whether the insertion of the functor into the loop has to be performed from below or from above respectively :

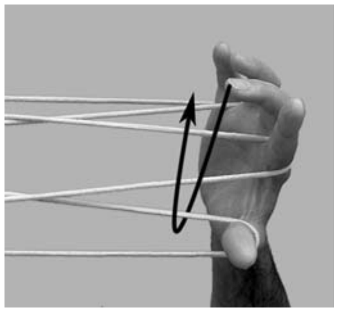
- $\underline{i}F\,(x\infty)$ means: insert the functor $F$ from below into the $x$ loop.

- $\bar{i}F\,(x\infty)$ means: insert the functor $F$ from above into the $x$ loop.

In Pictures 3a, the left thumb is inserted from above into the left index loop: $\bar{i}L1\,(L2\infty)$. Whereas in Picture 3b shows the right middle finger is inserted from below into the right thumb loop: $\underline{i}R3\,(R1\infty)$ :



3a- $L1$ inserted from above into $L2\infty$



3b- $R3$ inserted from below into $R1\infty$

**Convention**: The (elementary) operations implemented by the functors on the string are often performed simultaneously and symmetrically on both hands. When the mention of Left or Right ($L$ or $R$) is omitted, it will mean that the same operation has to be done simultaneously on both hands. In such a way, $\underline{i}1\,(2\infty)$ : means that both thumbs are inserted symmetrically from below into the index loops, considering that "1" indicates both right and left thumbs operating simultaneously and symmetrically.

**Remark**: For his Calculus for string figures, Storer introduces such functional notation, involving the notation $F(x)$, used to stress that it is Functor $F$ which operates on Object $x$. In our investigations into the topic, we have retained this principle of a functional language, but in adding a letter (in the previous case, "$i$" for the insertion) in order to precisely indicate the operation to be implemented. Storer uses instead a system of arrows : for instance, the previous insertion of the thumbs into the index loops is encoded $\underset{\rightarrow}{\underline{1}} \uparrow (2\infty)$.

The choice of noting each 'elementary operation' with a given letter has been made in order to facilitate a systematic study of the occurrence of the latter operations within a given (encoded) corpus of string figure (Vandendriessche, 2015a, p. 276-283), as it will be carried out as part of the current funded project "Encoding and Transmitting Knowledge with a String: a comparative study of the cultural uses of mathematical practices in string-figure making (Oceania, North & South America)" (ETKnoS, 2016-2020) that I am currently coordinating in collaboration with anthropologist Céline Petit[4].

[4]This research project, funded by the French National Research Agency (ANR), involves six researchers (ethnomathematicians, linguists, anthropologists). Through an ethnomathematical perspective, and based on ethnographies to be carried out in Oceanian, South and North American societies, the study aims particularly to investigate the relationships between the sequences of operations and the words spoken during the sequences. Our objective is to reach a better understanding of the extent to which the practice of string-figure making constitutes a method for organizing and transmitting knowledge (mythological, cosmological...), involving the use of mathematical concepts. On the Web : http://www.sphere.univ-paris-diderot.fr/spip.php?rubrique153&lang=en

### 2.4.2 Operation "Moving a functor over or under several strings"

When a given functor $F$ has to move either *over* or *under* several strings, we will adopt the following symbolism:

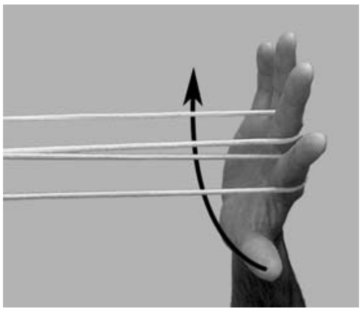
- $\overrightarrow{m}F(s)$ means: move the functor $F$ away over all strings up to and including the object $s$ (either string or loop).

- $\underrightarrow{m}F(s)$ means: move the functor $F$ away under all strings up to and including the object $s$.

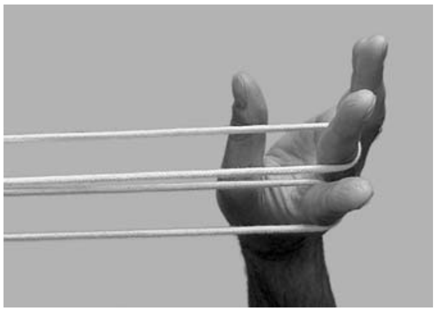
- $\overleftarrow{m}F(s)$ means: move the functor $F$ towards you over all strings up to and including the object $s$.

- $\underleftarrow{m}F(s)$ means: move the functor $F$ towards you under all strings up to and including the object $s$.

In pictures 4a and 4b, the right thumb $R1$ passes under all strings up to and including $R3f$. This operation is encoded $\underrightarrow{m}R1(R3f)$.



4a- $R1$ passing under all strings up to and including $R3f$



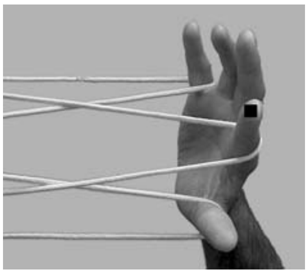
4b- Done

### 2.4.3 Operation "Releasing"

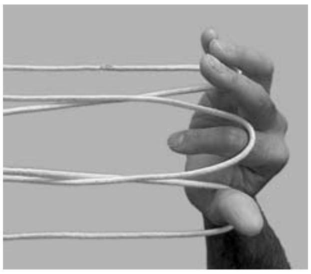
When the finger "$x$" of the right or left hand carries a single loop, the operation of "releasing" this loop is symbolized by $r\ x\ (x\infty)$, or $r(x\infty)$, considering implicitly that it is the functor "$x$" which operates.

In the example below, the right index releases its loop. This operation is encoded $r\ R2\ (R2\infty)$ or $r\ R2$.

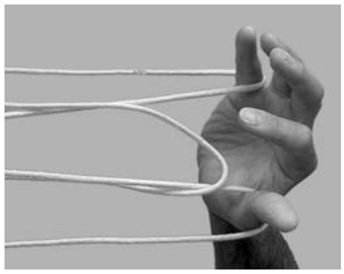
When the $i^{th}$ finger of a hand carries more than one loop, the release of a specific loop is written $r\ Ri\ (yRi\infty)$ or $r(yLi\infty)$ with $y \in l, m, u$. Moreover, the notation $r\ F$ will be used



5a- Releasing $R2\infty$



5b

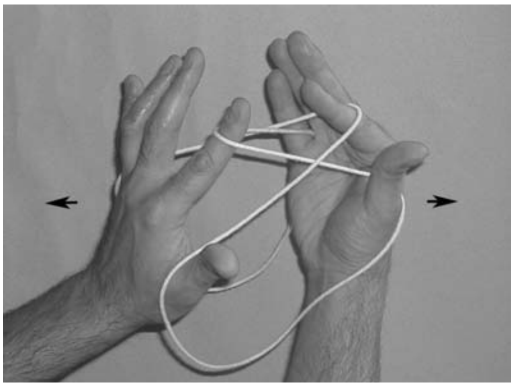


5c- Done

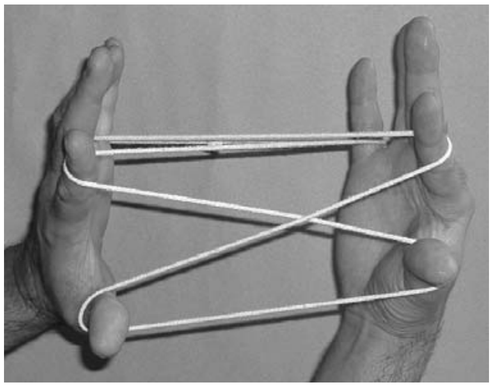
when a functor $F$ (including Teeth, Great toe, Wrist) has to release all of its loops. So, $r\ F$ and $r(F\infty)$ are equivalent when $F$ carries exactly one loop.

### 2.4.4 Operation "Extending"

The bar | indicates that the hands have to move apart in order to absorb the slack on the string. This movement corresponds to the (elementary) operation that I have called "extending" (the string) (Vandendriessche, 2007).



6a- Extending the string



6b- Done

We will use a double bar ||, when the string is extended, palms facing each other (cf. pictures 2 & 6b above). This operation is known in the literature as "Return to normal position".

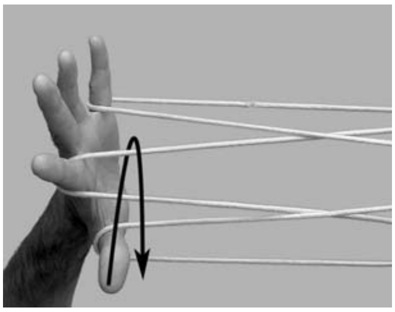
### 2.4.5 Elementary operation "Picking up"

The operation "Picking up" is encoded by the symbol "$p$":

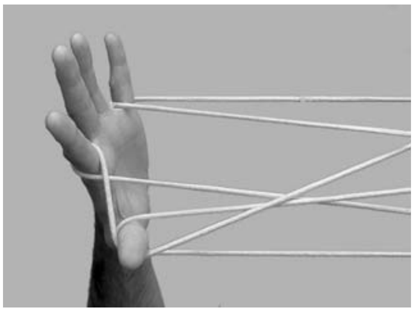
- $p\ F(s)$ means: $F$ (a finger) picks up the string $s$ from below, with the back of the finger.

An operation "Moving" generally precedes the operation "Picking up". Furthermore, the functor generally returns to position after having performed the latter operation. In the present symbolism, we will implicitly consider that the latter movement is performed after picking up a string.

In the example illustrated below, the left thumb $L1$ moves over both the left thumb far string $L1f$ and the left index near string $L2n$ and picks up from below the left index far string $L2f$. Then, the left thumb $L1$ returns to position (pictures 7a and 7b).



7a- $L1$ moves away from you over $L1f$ and $L2n$



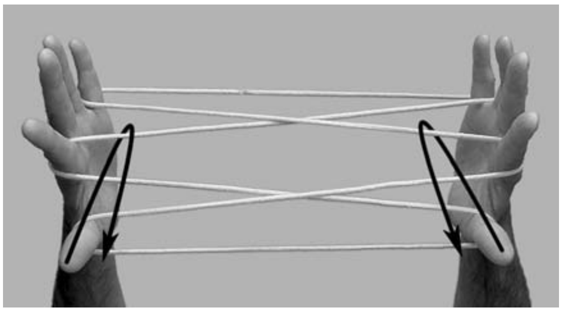
7b- $L1$ picks up $L2f$ and returns to position

This sequence is encoded: $\overrightarrow{m}L1\,(L2n) : p\ L1(L2f)$.

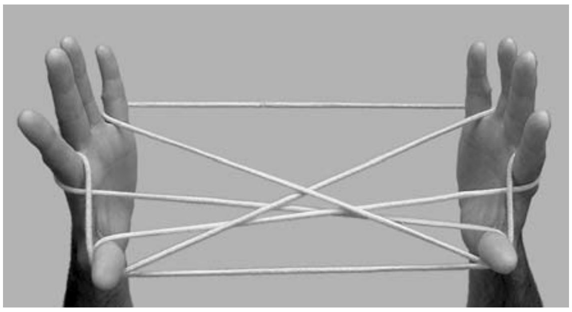
Like Storer, we use a colon to connect two consecutive operations.

As mentioned above, this operation "picking up" is frequently implemented simultaneously and symmetrically by the right and left hands. In pictures 8, the strings $R2f$ and $L2f$ are picked up symmetrically by the left and the right thumbs.

This sequence will be simply written as follows $\overrightarrow{m}1\,(2n) : p\ 1(2f)$.



8a- Move 1 away from you over $2n$ then pick up $2f$ and return to position



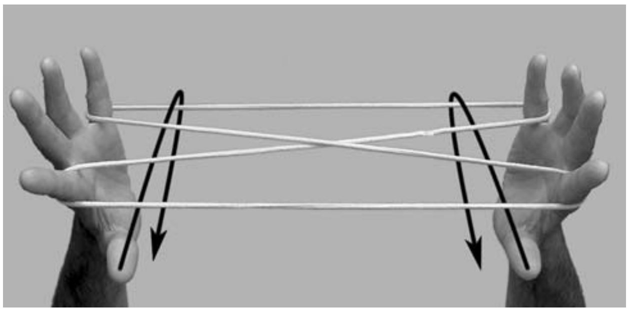
8b- Done

### 2.4.6 Operations "Hooking up" and "Hooking down"

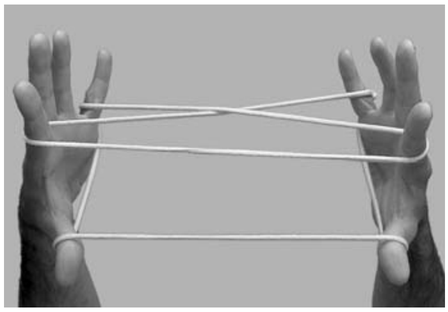
The operations "hooking" will be encoded by the symbol $h$, and either an "upper bar" or a "lower bar" to differentiate the operation "hooking up" from the operation "hooking down" respectively :

- $\underline{h}\ F\,(s)$ : means that the functor $F$ (a finger) hooks down the string $s$.

- $\overline{h}\ F\,(s)$ : means that the functor $F$ (a finger) hooks up the string $s$.



9a- 1 passing under 2 loops, hook up $5f$ and return to position

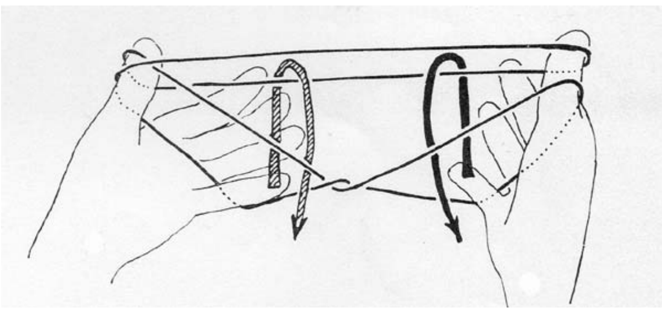


9b- Done

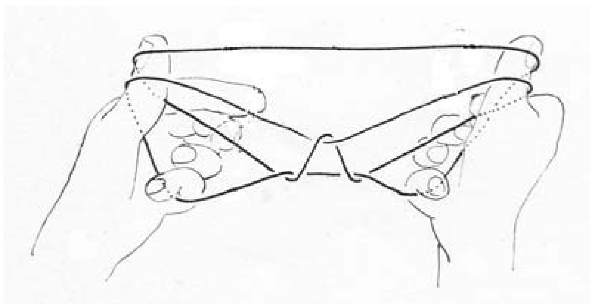
In this example illustrated by the pictures 9a-9b, the operation "hooking up" is preceded by the operation "moving a functor under several strings" described above. The sequence is thus encoded as : $\underset{\rightarrow}{\underline{m}}1(5n) : \overline{h}\ 1\,(5f)$

In the pictures 9c-9d, the operation "hooking down" is implemented on the $1f$ strings by the little fingers: this is encoded $\underline{h}\ 5\,(1f)$.

**Remark**: After performing the operation "hooking down", the finger stays where it is, without returning to its initial position. If necessary, when not implicit, we will use the symbol # to emphasize this.



9c- 5 hooking down $1f$



9d- Pictures adapted from (Victor, 1940)

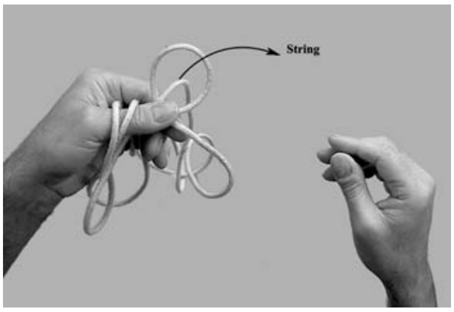
### 2.4.7 Operations "Seizing" and "Grasping"

**Seizing an object** A string can be seized between either two fingers or the teeth. We will denote this elementary operation by using the letter "$s$".

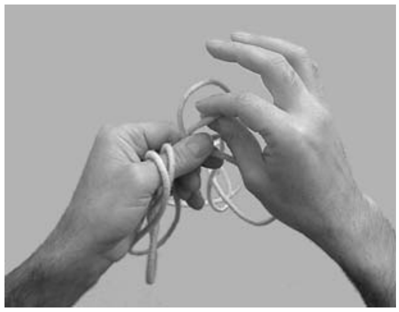
- $s\ L\underline{x}\overline{y}(s)$ means: seize the string $s$ between the left fingers $x$ (going under "$s$") and $y$ (going above "$s$").

- $s\ L\overline{x}\underline{y}(s)$ means: seize the string $s$ between the left fingers $x$ (going above "$s$") and $y$ (going under "$s$").

In the pictures 10a-10b below, the string $L2f$ is seized between $R1$ and $R2$. It will thus be encoded: $s\ R\underline{1}\overline{2}(L2f)$.

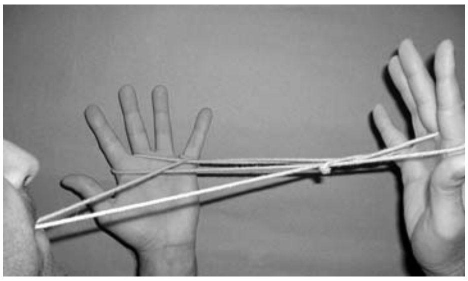



10a



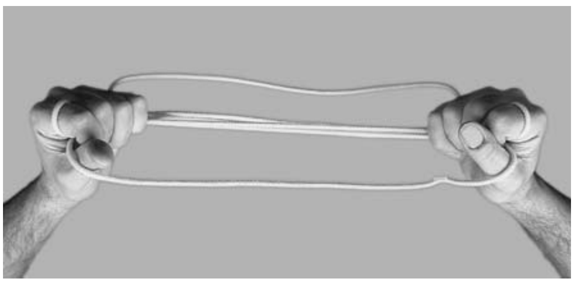
10b- Seizing a string with $R1$ and $R2$

When a string is seized with the teeth. It will be encoded $s^*\ T(s)$. In the picture 10c below the teeth have seized the string $5f$: this is encoded $s^*\ T(5f)$.



10c-Seizing a string with the teeth

**Grasping several strings** It happens that several strings or loops are grasped simultaneously by several fingers. We will use the letter "$g$" to denote this operation. In the example illustrated in the picture 10d the fingers 2345 (of both hands) grasp simultaneously the string $1f$ and the loops $2\infty$ and the string $5n$). This is encoded: $g\ 2345(1f, 2\infty, 5n)$.



10d- Grasping several strings

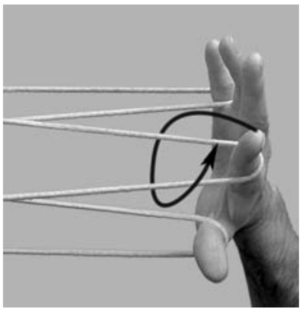
### 2.4.8 Operation "twisting" a finger (or a loop)

A finger can be rotated in order to close the loop carried by this finger *i.e.* to make its two strings cross each other.

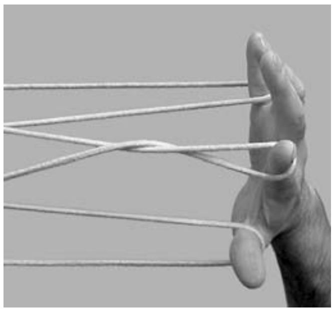
- $tw^+x$ means: rotate the finger $x$, 180° anticlockwise—for an observer located on the left side of the practitioner.

- $tw^-x$ means: rotate the finger $x$, 180° clockwise—for an observer located on the left side of the practitioner.

For instance, pictures 11 illustrate the operation $tw^+R2$:



11a- Twisting R2



11b- Done

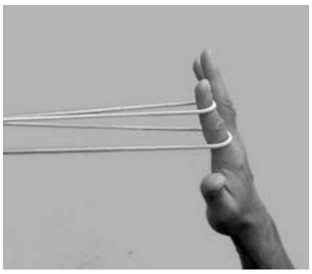
### 2.4.9 Rotations

A horizontal rotation of a functor $F$ (generally the hands or wrists) will be encoded by using the symbol $hr$. More precisely,

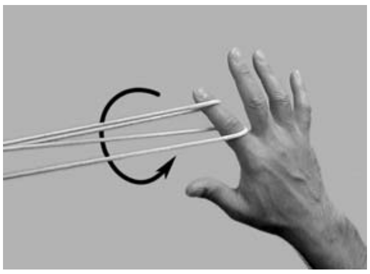
- $hr^+F$ if the rotation is performed 360° anticlockwise—for an observer located on the left side of the practitioner.

- $hr^-F$ if the rotation is performed 360° clockwise—for an observer located on the left side of the practitioner.

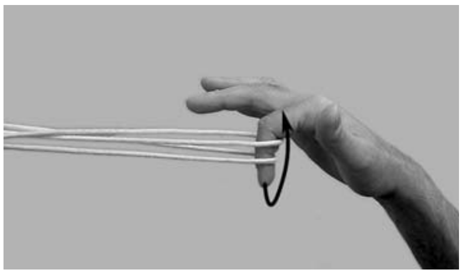
The pictures 12a-12c show the horizontal rotation of the right hand (or wrist) 360° anticlockwise. This is encoded $hr^+RH$.



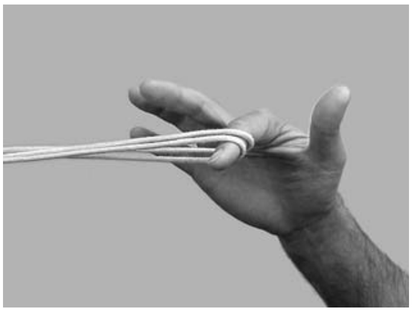
12a



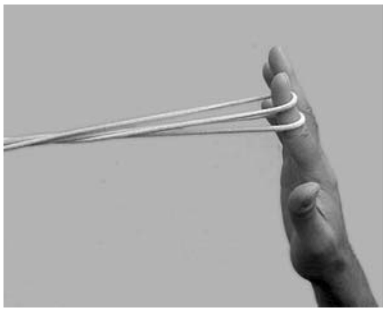
12b



12c



12d



12e

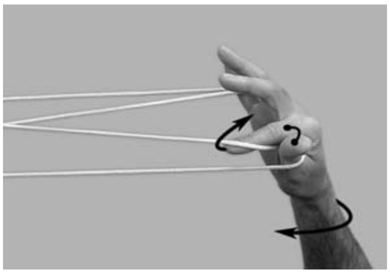
**Remark**: when a horizontal rotation is implemented with an angle less than 360°, we will encode this using a fraction. For instance, the horizontal rotation of the right hand (or wrist) 90° anticlockwise will be encoded: $\frac{1}{4}hr^{+}RH$. See the example in section 3.2.

A vertical rotation of a functor $F$ (generally the hands or wrist) will be encoded by using the symbol $vr$. More precisely,

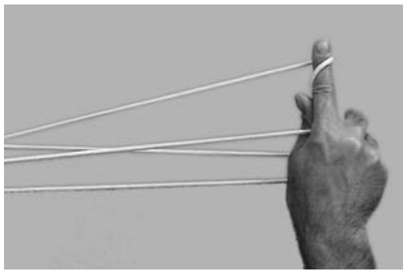
- $vr^{+}H$ means: rotate both hands anticlockwise for an observer located underneath the practitioner's hands pointing up *i.e.* positioning the palms away.

- $vr^{-}H$ means: rotate both hands clockwise for an observer located underneath the practitioner's hands pointing up *i.e.* positioning the palms towards the body.

A vertical rotation $vr^{+}H$ occurs on the right hand, for instance, at the end of the "Caroline extension" (cf. section 2.5.4) (pictures 12f-12g).



12f



12g

### 2.4.10 Summary

| Summary table | | |
|---|---|---|
| **Operations** | **Symbols** | **Description** |
| Inserting | $\bar{i}F\,(x\infty)$ / $\underline{i}F\,(x\infty)$ | $F$ is inserted from above/below into $x\infty$ |
| Moving | $\overrightarrow{m}F(s)$ / $\underrightarrow{m}F(s)$<br>$\overleftarrow{m}F(s)$ / $\underleftarrow{m}F(s)$ | pass $F$ "away/towards you" "over/under"<br>all strings up to and including the string $s$ |
| Releasing | $r\ x(x\infty)\ or\ r(x\infty)$<br>$r\ F$ | Release both loops $x\infty$<br>Release all the loops on the functor $F$ |
| Extending | $\vert$ | Extend the string, pulling hands apart |
| Picking up | $p\ F(s)$ | $F$ picks up the string $s$ |
| Hooking up | $\bar{h}\ F\,(s)$ | $F$ hooks up the string $s$ |
| Hooking down | $\underline{h}\ F\,(s)$ | $F$ hooks down the string $s$ |
| Seizing | $s\ L\underline{x}\overline{y}(s)$ / $s\ L\overline{x}\underline{y}(s)$ | Seize the string $s$ between the left finger $x$ and $y$<br>$x$ going under, and $y$ going over "$s$"<br>or vice versa |
| Seizing with the teeth | $s^*\ T(x)$ | $T$ seize the object $x$ |
| Grasping<br>with the fingers $x$,$y$, and $z$ | $g\ xyz(s_1, s_2, s_3)$ | $F$ grab the strings (or loops) $s_1, s_2, s_3$ |
| Twisting a finger | $tw^+x$ / $tw^-x$ | Rotate the finger $x$,<br>180° anticlockwise/clockwise<br>for an observer located on the left side<br>of the practitioner |
| Horizontal rotation | $hr^+F/hr^-F$ | Rotate the hand/wrist<br>anticlockwise/clockwise<br>for an observer located on<br>the left side of the practitioner |
| Vertical rotation | $vr^+F/vr^-F$ | Rotate one hand anticlockwise/clockwise,<br>for an observer located underneath the<br>practitioner's hands, fingers pointing up |
| Not returning | $\#$ | After performing an operation<br>keeping the functor in its place |

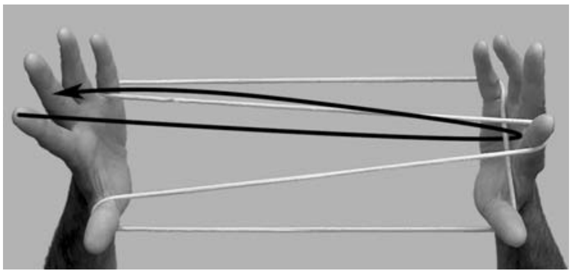
| | | |
|---|---|---|
| Restoring | $\|\|$ | Place the hands,<br>palms facing one another, fingers pointing up |

## 2.5 Encoding Sub-Procedures

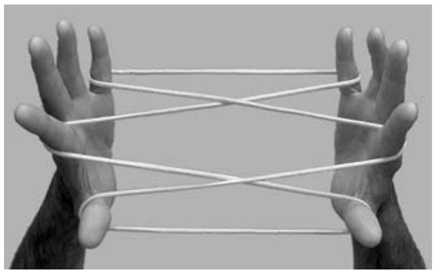
### 2.5.1 Openings

An Opening can be defined as a sub-procedure aiming at creating a certain number of loops on the hands, starting from the original loop of string. The openings are noted $\underline{O}$. For instance, Opening $A$ is encoded $\underline{O}.A$ (pictures 13a-13c).

13a



13b



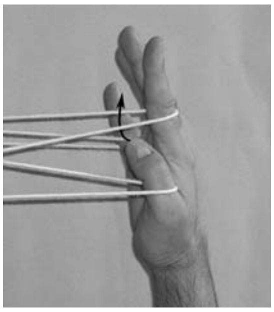
13c

### 2.5.2 Transfer

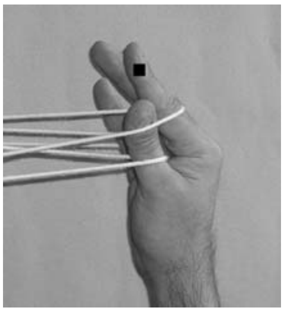
Transferring the loop carried by the finger $x$ to the finger $y$ consists in inserting the finger $y$, either from above or below into the loop of the finger $x$, then releasing the latter finger $x$. In the first case, we will say that the loop $x\infty$ is distally transferred to $y$, whereas in the second case, it is proximally transferred to $y$.

- $\underline{TF}\,(y, x\infty)$ means: transfer proximally $x\infty$ to $y$.
- $\overline{TF}\,(y, x\infty)$ means: transfer distally $x\infty$ to $y$.

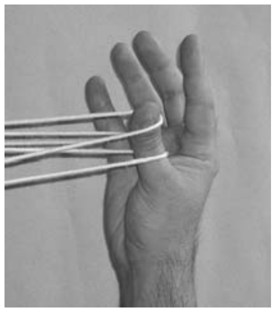
In pictures 14a-14c, it is the right index loop which is proximally transferred to the thumb. This will be encoded : $\underline{TF}\,(R1, R2\infty)$.



14a- Inserting



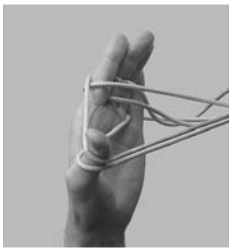
14b- Releasing



14c- Transferring

### 2.5.3 Navajo

When two loops lie on the same finger $x$ (picture 15a, left thumb) the "Navajo" operation is implemented on this finger by passing the proximal loop over the distal one, and then, over the fingertip where it is released (pictures 15b-15c). We use the term "Navajo" as a verb, saying "Navajo finger $x$". Finally, it is encoded $N\ x$.



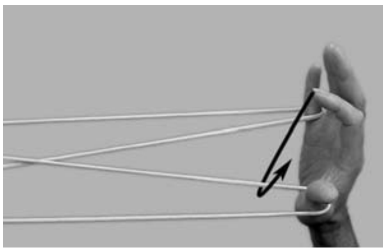
15a

15b

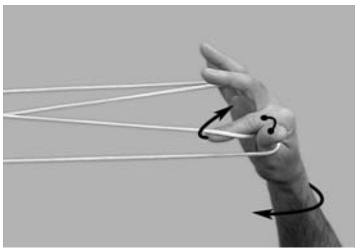
15c- Navajo

### 2.5.4 Caroline Extension

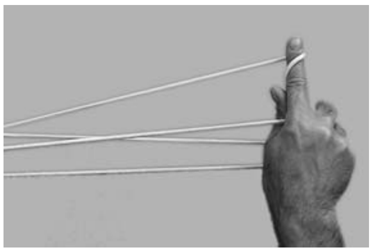
When a thumb carries a loop, the Caroline extension consists in picking up the far thumb string (picture 16a), while pressing the thumb against the index in order to seize the latter string (picture 16b), and, finally, rotating the hands outwards (pictures 16b and 16c). This sub-procedure is encoded $\underline{E}.C$.



16a- Picking up



16b- Seizing and rotating



16c- Caroline extension

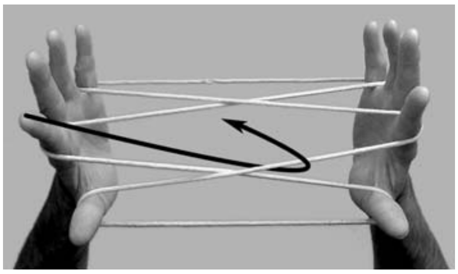
### 2.5.5 Exchange

The sub-procedure "Exchanging two loops" consists in exchanging a loop with the same loop on the opposite hand, after passing one of these loops either from above into the other.

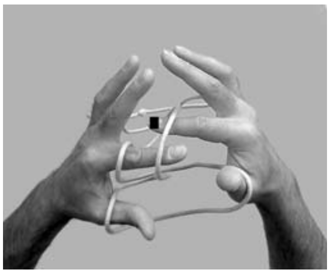
- $EX\ x\infty\ [R > L]$ means: exchange the $x^{th}$ loops, inserting (from above) the right loop into the left one.

- $EX\ x\infty\ [L > R]$ means: exchange the $x^{th}$ loops, inserting (from above) the left loop into the right one.

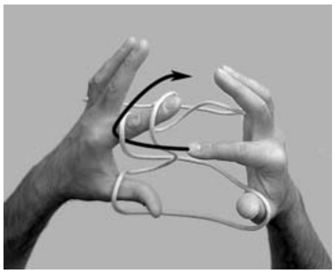
Pictures 17a to 17e show the "exchange" of the index loops, $R2\infty$ passing into $L2\infty$: this will be encoded $EX\ 2\infty\ [R > L]$.



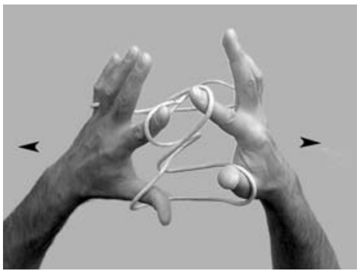
17a



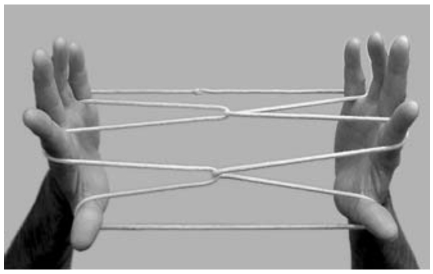
17b



17c



17d

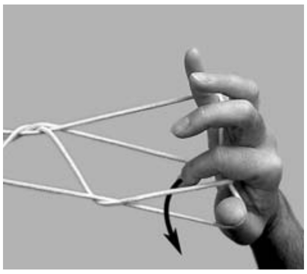


17e

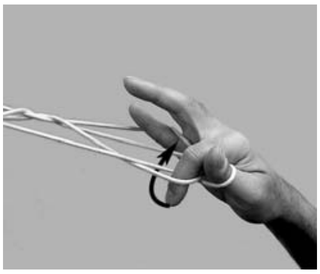
### 2.5.6 Twist of a finger

The "twist of a finger"—is a succession of two elementary operations implemented by a finger (generally the indices). The finger $x$ first picks up a string $s_1$ and second, hooks up the string $s_2$. This will be encoded $TW\ x(s_1, s_2)$.

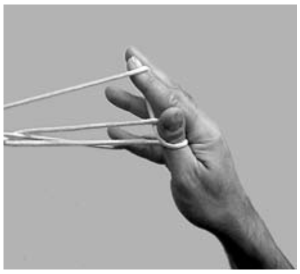
Pictures 18a to 18c show the Twist of the right index, picking up a diagonal string $s$, then hooking up $R1n$. This will be encoded $TW\ R2(s, R1n)$. Note that the picked up diagonal string is released as the right index returns to an upright position.



18a



18b



18c

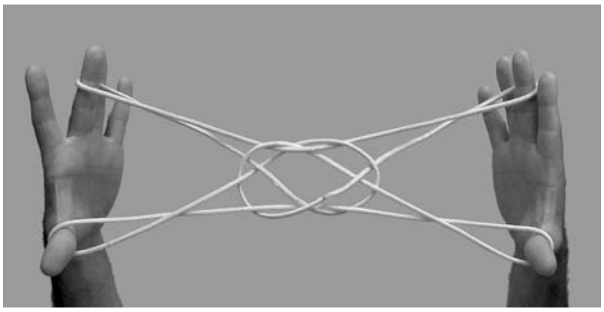
### 2.5.7 Summary

| Summary table | | |
|---|---|---|
| **Sub-procedures** | **Symbols** | **Description** |
| Openings | $\underline{O}$ | $\underline{O}.A$ means Opening A |
| Transfer | $\underline{TF}\,(y, x\infty)\ /\ \overline{TF}\,(y, x\infty)$ | Proximally (resp. distally) transfer $x\infty$ to $y$ |
| Navajoing | $N\ x$ | Navajo the finger $x$ |
| Caroline extension | $\underline{E}.C$ | Perform the Caroline extension |
| Exchanging | $EX\ x\infty\ [R > L]\ /\ EX\ x\infty\ [L > R]$ | Exchange $Rx\infty$ and $Lx\infty$ passing $Rx\infty$ into $Lx\infty$ (from above) or the reverse |
| Twisting | $TW\ x(s_1, s_2)$ | $x$ picks up $s_1$ then hooks up $s_2$ |

# 3 Examples

## 3.1 Niu (sun), Solomon Islands

$\underline{O}.A\ :\ \bar{i}1\,(2\infty)\ :\ p1\,(2f)\ :\ \overleftarrow{m}3(2n)\ :\ \underline{i}3\,(l1\infty)\ :\ p3\,(l1f)\ :\ r1\ |\ \bar{i}1\,(2\infty)\ :\ \underrightarrow{m}1(3\infty)\ :\ \underline{i}1\,(5\infty)\ :$
$p1\,(5n) : r5 : r2\ |$



19- Niu (sun)

## 3.2 Katagjuk (the entrance), Iglulik, Eastern Canada, Arctic

$\underline{O}.A : \overrightarrow{m}2\,(5\infty) : \overline{h}2\,(2f, 5\infty) : \underline{i}2\,(1\infty) : \overline{h}2\,(1n) : r1 : \underline{h}1\,(l2n) : \underset{\rightarrow}{m}1\,(5n) : p1\,(5f) : \underline{h}1\,(u2n) : r\,(ex.1\infty)^{*} : \underset{\rightarrow}{m}1\,(2\infty^{2}, 5f) : \overrightarrow{m}1\,(5n) : p1\,(5n) : \underset{\rightarrow}{m}1\,(ltv) : p1\,(ltv) : r2 \;||\; \frac{1}{4}hr^{+}H$

(*) while the operation $\underline{h}1(u2n)$ is implemented the loop previously carried by the thumbs (noted $ex.1\infty$) are finally released.

20- Katagjuk (the entrance) © Céline Petit

## 3.3 Dakuna (Magic Stones), Trobriand Islands

**- Figure 1**

$\underline{O}.A : \overrightarrow{m}1(2\infty) : \underline{i}1(5\infty) : p1(5n) : \overrightarrow{m}2(u1f) : \underline{i}2\,(l1\infty) : p2\,(l1f) : r1 \mid \bar{i}1\,(u2\infty) : p1\,(u2f) : \overleftarrow{m}5(l2\infty) : p5(u2n) : r(u2\infty) \mid \overrightarrow{m}2(u5n) : \underline{i}2\,(l5\infty) : p2\,(l5n) : r5 \mid \underline{TF}\,(5, u2\infty) : \overrightarrow{m}1(2\infty) : \underline{i}1\,(5\infty) : p1\,(5n) : \underline{E}.C : r2 : \underline{E}.C \mid$

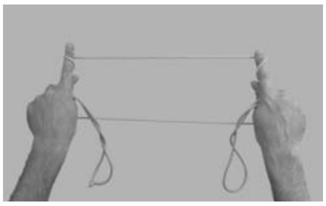
**- Figure 2**

$\underline{O}.A : \overrightarrow{m}1(2\infty) : \underline{i}1(5\infty) : p1(5n) : \overrightarrow{m}2(u1f) : \underline{i}2\,(l1\infty) : p2\,(l1f) : r1 \mid \left\{ \begin{array}{c} \bar{i}L1\,(Lu2\infty) : pL1\,(Lu2f) \\ \overrightarrow{m}R1(Rl2\infty) : pR1\,(Ru2f) \end{array} \right\}^{*} \overleftarrow{m}5(l2\infty) : p5(u2n) : r(u2\infty) \mid \overrightarrow{m}2(u5n) : \underline{i}2\,(l5\infty) : p2\,(l5n) : r5 \mid \underline{TF}\,(5, u2\infty) : \overrightarrow{m}1(2\infty) : \underline{i}1\,(5\infty) : p1\,(5n) : \underline{E}.C : r2 : \underline{E}.C \mid$

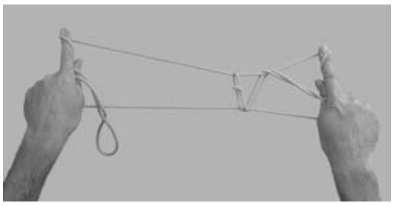
(*) we use such a column bracket to encode sequences of operations that can be theoretically performed simultaneously, but can also be performed one hand after the other.

**- Figure 3**

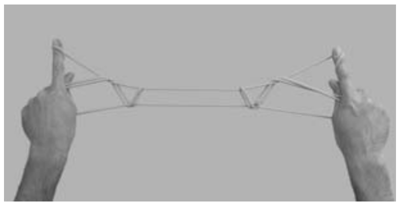
$\underline{O}.A : \overrightarrow{m}1(2\infty) : \underline{i}1(5\infty) : p1(5n) : \overrightarrow{m}2(u1f) : \underline{i}2\,(l1\infty) : p2\,(l1f) : r1 \mid \overrightarrow{m}1(l2\infty) : p1(u2f) : \overleftarrow{m}5(l2\infty) : p5(u2n) : r(u2\infty) \mid \overrightarrow{m}2(u5n) : \underline{i}2\,(l5\infty) : p2\,(l5n) : r5 \mid \underline{TF}\,(5, u2\infty) : \overrightarrow{m}1(2\infty) : \underline{i}1\,(5\infty) : p1\,(5n) : \underline{E}.C : r2 : \underline{E}.C \mid$



21a- *Dakuna 1*: *dukuyoyo wa* (they fly away)



21b- *Dakuna 2*: *dukuyoyo - lukutota* (they fly away - they remain standing)



21c- *Dakuna 3*: *dukutota* (they remain standing)

# 4 In conclusion

The symbolic writing system presented in this paper has been devised in order to undertake a comparative formal analysis of different string figure corpora—at the level of the elementary operations, as well as the sub-procedures, both involved in string figure processes. This comparative approach is currently applied to the formal comparison of different string figure corpora from Oceania and North & South America collected thanks to the current funded project ET-KnoS (cf. footnote 3), and will lead to upcoming publications. Obviously, the encoding system used for the latter study will be constantly refined and improved. In particular, new symbols should be created to allow the coding of the upcoming collated data.

As is generally the case for symbolic notations—developed by mathematicians—, this new coding system contains a form of ambiguity i.e. different ways of encoding the same sequence of operations can be equally efficient. For example, the sequence "Opening A, insert 1 from above into 2 loops, pick up 2f and return", can be coded either by :
$\underline{O}.A : \bar{i}1(2\infty) : p1(2f)$ or $\underline{O}.A : \overrightarrow{m}1(2n) : p1(2f)$.

However, this could actually be analyzed as a "productive ambiguity"—to borrow American philosopher Emily Grosholz's terminology (Grosholz, 2007)—, leading, for instance, to mathematical equivalence between different coding sequences, as well as to the elaboration of a genuine calculus, searching its own rules and properties.

Furthermore, everyone who has attempted to encode gestures and/or movements knows that it is sometimes not easy to avoid using natural language. Once again, this should not be seen as a bad practice when coding string figure processes. De facto, the use of natural language constantly occurred in mathematical/formalization practices. Indeed, the articulation between symbolic writings with natural language actually produces interactions that may sometimes allow new ideas to arise (Grosholz, 2007).

Finally, as mathematician Ali R. Amir-Moez (1919-2007) suggests in his book *Mathematics* *and string figures* (Amir-Moez, 1965), such a symbolic writing system should have an impact on the development of the string figure-making practice itself, using the latter system to create new procedures and figures.

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