Rethinking the history of mathematical symbolism
A cooperative project

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General histories of mathematics commonly take up the view that mathematical symbolism was an innovation introduced in Europe at the end of the 16th century. The project aims to explore the hypothesis that what we have at the time is more precisely a new type of mathematical symbolism, but that it draws on previous types of symbolism. In other words, the project will suggest rethinking what we mean by "symbolism". In particular, we aim to explore the hypothesis that mathematical symbolism has a much more global history than what has been so far assumed.

In its first year, the project has begun with a historical exploration of the historiographies of mathematical symbolism through a seminar and a conference (https://edin.ac/4aqArkm.) These two types of events have two main aims.

The first is precisely to explore the historical shaping of the view that mathematical symbolism originated with 16th- and 17th century European actors. The historical importance given to their work in this respect usually derives from their use of literal computation. How did different historians and philosophers understand the specificities and the virtues of this type of computation? What facets of symbolism have been emphasized in relation to the claim that mathematical symbolism was a European invention? And also, what facets of symbolism have remained overshadowed, or been treated as deriving from properties of symbolism perceived as primary?

Our second goal is to examine other properties and virtues of mathematical symbolism that other actors have foregrounded in their historical analysis and how these different starting points led them to consider other notations as falling within the purview of a history of mathematical symbolism. A prominent example of this kind is Charles Burnett's work on decimal place-value notation (Burnett 2002). In it, Burnett emphasizes that this notation was independent from the spoken language just as literal notation is, and he suggests that this feature is correlated with a circulation of the notation across linguistic borders, similar to that of literal notation. As a result, Burnett invites us to consider decimal place-value notations as belonging to a history of mathematical symbolism.

The reflections that we have developed along the first year will be pursued in the coming years, during which we will concentrate on facets of mathematical symbolism that have appeared as promising to us as what Burnett has emphasized, but that have not yet received the attention they deserve. In particular, in the second and third years of the project, we expect to explore the assumption that two such facets include:

— the diagrammatic properties of the symbolic notations with the practices of navigation that these notations require when manipulated mathematically (year 2)

as well as

— the practices of formal work in mathematics (year 3).

The study of the **diagrammatic dimensions of mathematical notations** and/or inscriptions appears to be promising for the project inasmuch that these dimensions will allow us to examine practices that assume a distance between, on the one hand, oral speech and, on the other, notations and/or inscriptions. Moreover, their study brings into focus the shaping of navigation through notations and/or inscriptions, which is a key dimension of symbolic work. Giaquinto (2007) has explored the diagrammatic dimensions of modern symbolic notations, analysing how they are used in mathematical work. What is the history of these diagrammatic features and of the practices that bring them into play? Are they related to the tabular types of computation that we find in, for instance, Sanskrit sources (Keller, Montelle, and Koolakudlu forthcoming)? These are questions that will be addressed on the basis of sources in, e.g., cuneiform, Chinese, Sanskrit, Arabic, and Latin, which abound in notations and computations with such diagrammatic features. Indeed, whether we look at Chinese sources such as Qin Jiushao's 秦九韶 *Mathematical Writings in Nine Chapters* (數書九章, 1247) or at Ibn al-Yāsamīn's *Grafting of Opinions of the Work on Dust Figures*, composed in Maghreb in the twelfth century (Oaks 2007), the notation of numbers and computations with place-value decimal notations and that of equations and of the process determining their roots were both viewed as "figures," and mathematical work with them put into play similar diagrammatic features. These remarks offer a perspective from which to examine the relationship between these features as practiced with numbers written with a place-value notation and as practiced with other types of notations and/or inscriptions. The research devoted to the diagrammatic features of mathematical notations and/or inscriptions in ancient and medieval sources will examine whether such features are essentially associated with practices of computation or whether they can be encountered in other types of mathematical practice. If they prove to be attached to computation, we will need to spell out the properties that these diagrammatic features impart to the practices of computation that rely on such notations and/or inscriptions. In particular, a key question will be to understand the relationship that can be established between the deployment of notations and/or inscriptions with diagrammatic features and the use of place-value notations.

Symbolic notations have been associated with **formal work in mathematics**. The third year will concentrate on the history of such type of work and, notably, on the history and the practices of formal computations in ancient and medieval mathematical sources. In particular, as we have begun to show in (Chemla in dialogue with A. Keller and C. Proust 2022) and in (Keller and Morice-Singh 2022), one key feature of the work with place-value notations is that, just as with modern symbolic notation, it can be formal. However, what does formal mean in these contexts, and how was it articulated with other practices of formal work with operations as embodied in the writing of some algorithms (Chemla 1992)? Can we identify other types of formal practices in ancient and medieval mathematics? How do these sources help us better understand what is at stake with formal practices in mathematics? These will be some of the questions that will be at the horizon of this third year. The key point will in particular be to understand the relationship between the development of formal computations and the history of mathematical symbolism. We will also examine whether authors of the sources considered give us clues to their own ideas about notations, symbolism, and formal facets of their practices, including diagrammatic practices.

On the basis of the explorations carried out in the first three years, the fourth year will aim at offering a **new perspective on the history of mathematical symbolism**. One of its goals is to rethink the relationship between the various features that we conventionally associate with mathematical symbolism, thereby opening the possibility of a historical approach that would be devoted to each of these features separately. Another goal is to explore in particular the reasons why we might consider some practices with place-value numeration systems as belonging to the history of mathematical symbolism.

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